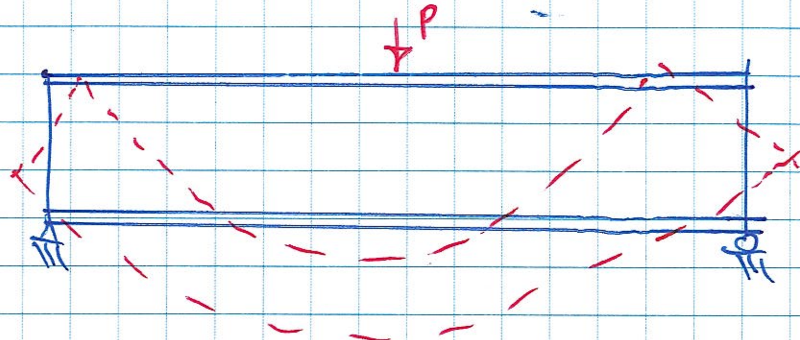
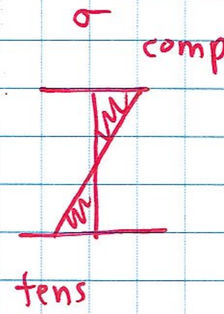
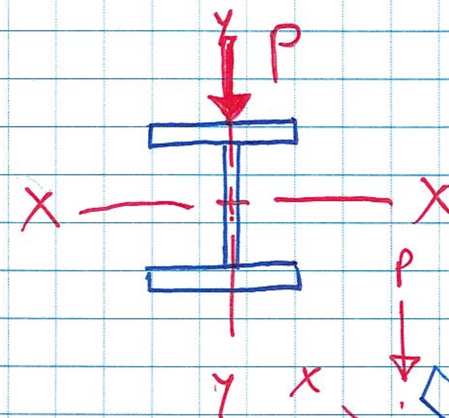
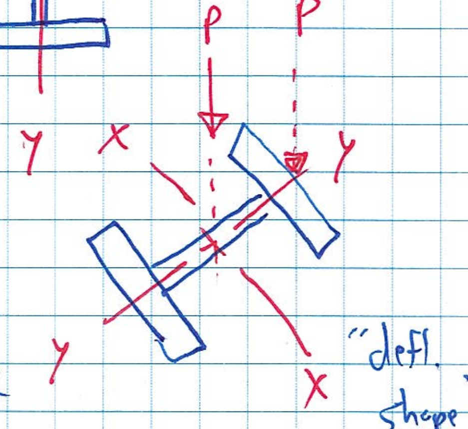
FLEXURE - STABILITY OF BEAM









- COMP. Flange (top) wants to buckle laterally about the weak-axis <u>BUT</u> its restrained by the tension flange

⇒ Beam deflect down (laterally) & Twist (Torsion)

This is called lateral-torsional buckling (LTB)

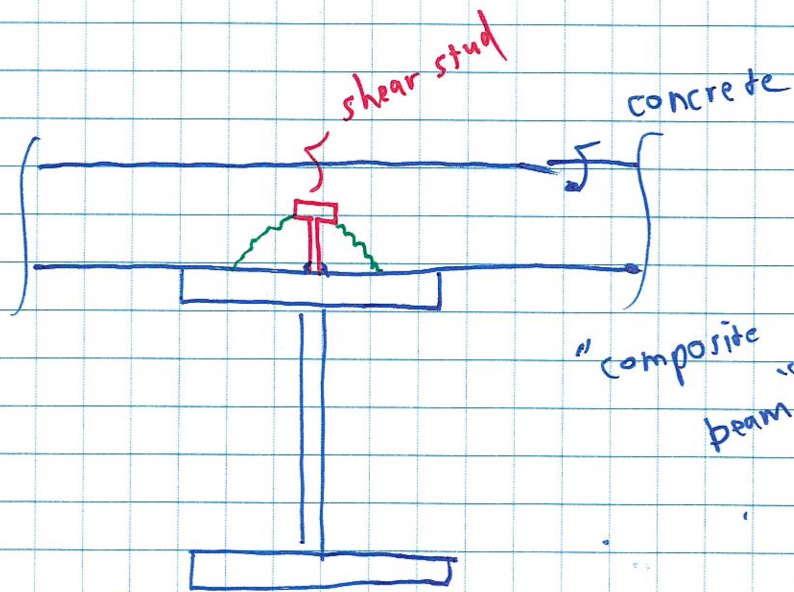
f(unbraced length, moment gradient)

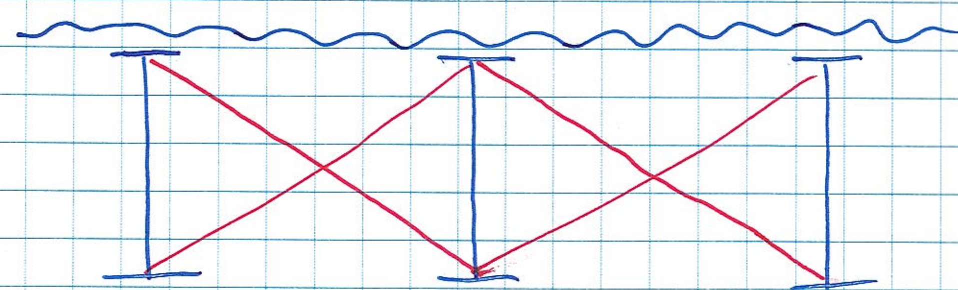
- If the comp. flange is "fully braced" against LTB, the moment capacity is $M_n = M_p = f_y Z_x$ ← plastic section Modulus

plastic moment

• If not "fully braced", the capacity is controlled by LTB

Elastic Inelastic





OTHER STABILITY ISSUES: FLANGE LOCAL BUCKLING (FLB)
WEB LOCAL BUCKLING (WLB)

CLASS. OF SHAPE

AISC CHAP B, Table B4.1b, p 16.1 - 18 & 19

"flange" CASE 10: $\frac{b_f}{2t_f} \leq 0.38\sqrt{\frac{E}{f_y}} = \frac{65}{\sqrt{f_y}}$

"web" CASE 15: $\frac{h}{t_w} \leq 3.76\sqrt{\frac{E}{f_y}} = \frac{640}{\sqrt{f_y}}$

"compact" shape → no FLB or WLB for I-shaped members

BENDING CAPACITY OF COMPACT SHAPES

AISC F2, pg 16.1-47 "W-Shapes, Strong-Axis, Compact"

- For $L_b \leq L_p$

$$M_n = M_p = F_y Z_x \qquad (F2\text{-}1)$$

"Basic Strength / Plastic Moment"

- For $L_p < L_b \leq L_r$

$$M_n = C_b\left[M_p - \underbrace{\frac{\left(M_p - 0.7F_y S_x\right)}{(L_r - L_p)}}_{BF}(L_b - L_p)\right] \leq M_p \qquad (F2\text{-}2)$$

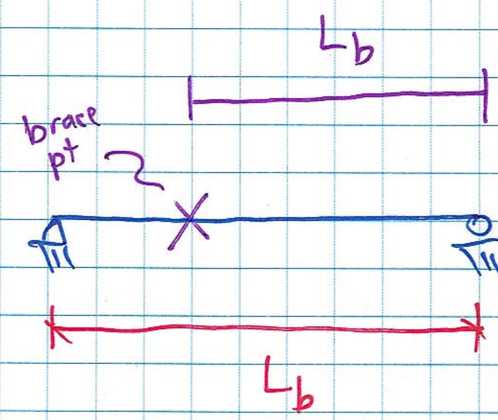
BF "Beam Factor" "Inelastic LTB"

$$M_n = C_b\left[M_p - BF(L_b - L_p)\right] \leq M_p$$

$$\phi_b M_n = C_b\left[\phi_b M_p - \phi_b BF(L_b - L_p)\right] \leq \phi_b M_p \quad \leftarrow \text{KEY EQN}$$

- For $L_b > L_r$ "Elastic LTB"

$$M_n = F_{cr} S_x \le M_p \qquad \text{(F2-3)}$$



where: L_b = unbraced length

$I_y, J, S_x, h_o, c_w, r_{ts}$ ← Sect. Prop

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}}\right)^2} \qquad \text{(F2-4)}$$

$$L_r = 1.95\, r_{ts} \left(\frac{E}{0.7 F_y}\right) \sqrt{\frac{Jc}{S_x h_o}} \sqrt{1 + \sqrt{1 + 6.76 \left(0.7 \left(\frac{F_y}{E}\right)\left(\frac{S_x h_o}{Jc}\right)\right)^2}}$$

"Reduced Form" (F2-6)

$$L_p = \frac{300\, r_y}{\sqrt{F_y}}$$

in.

ksi

*E = 29000 ksi

c = 1.0 "for all W-shapes"

h_o = distance between centroid of the flanges = $d - t_f$

- and -

$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3M_A + 4M_B + 5M_C} \quad \text{(F1-1)}$$

allows for an increase in LTB strength for non-uniform moment gradient over an unbraced length

where:

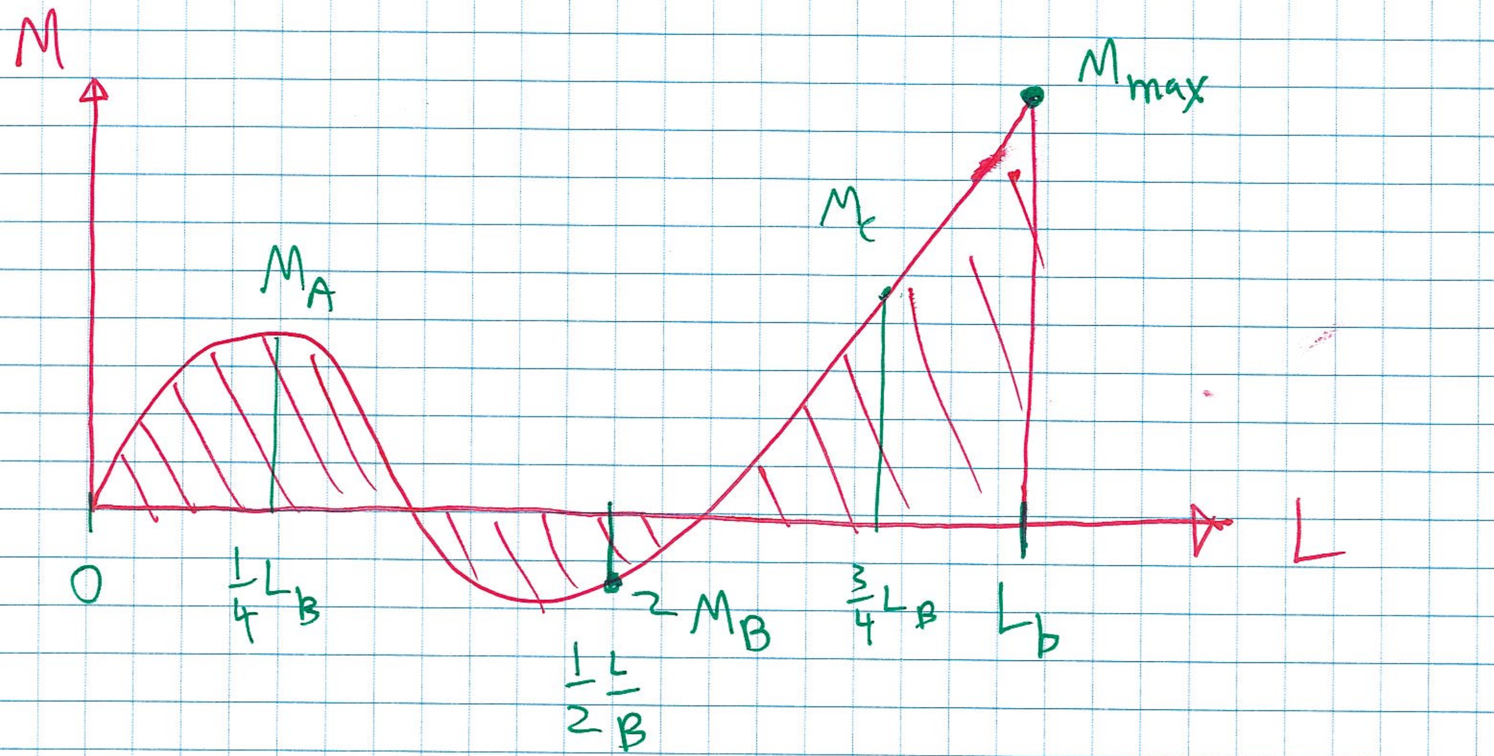
M_{max} = Max. Moment in the unbraced length

M_A = Mom. @ $\frac{1}{4}L$ in the " "

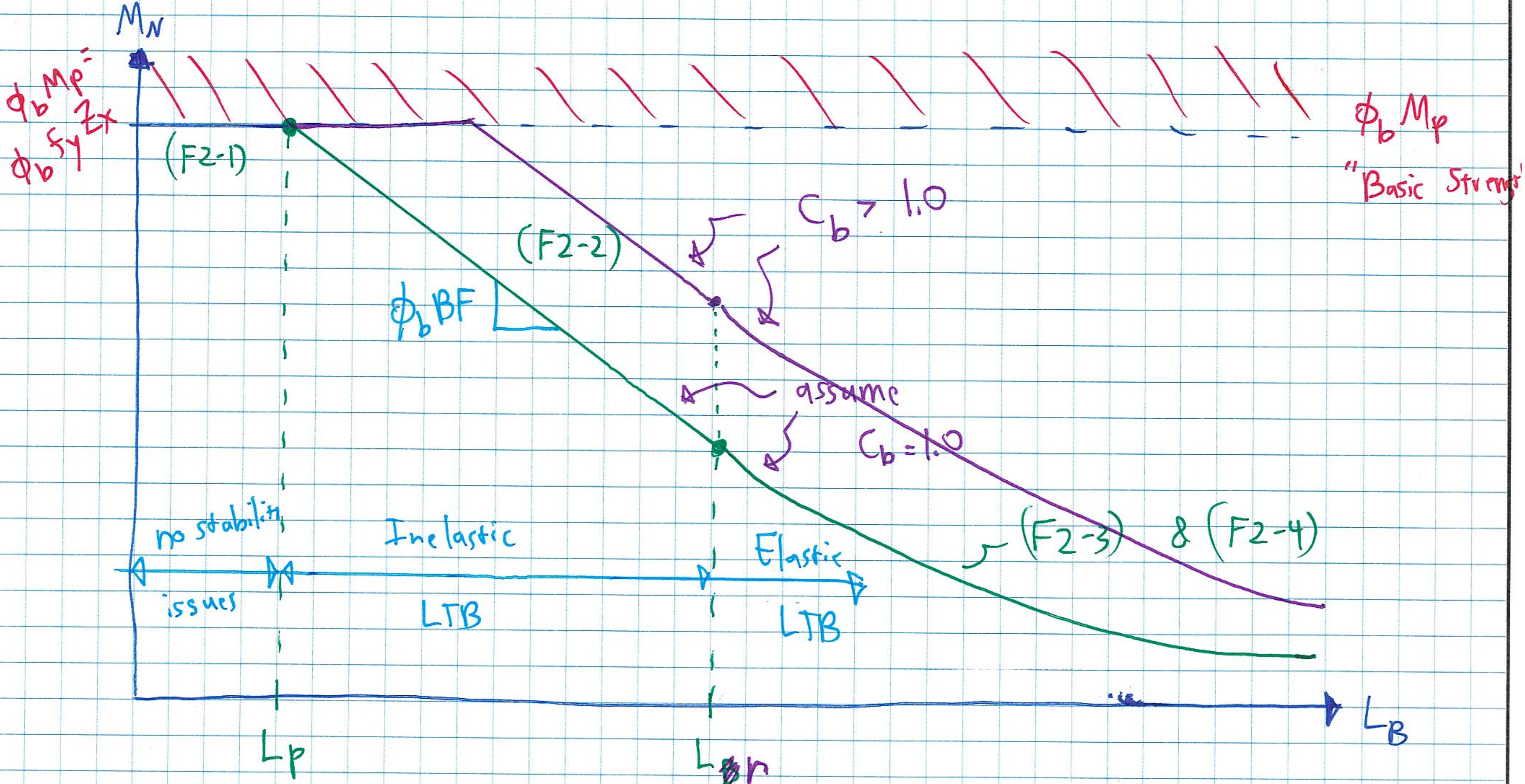
M_B = " @ $\frac{1}{2}L$ " " " "

M_C = " @ $\frac{3}{4}L$ " " " "

$C_b = 1.0$ for constant moment or unbraced cantilevers


M
M_{max}
M_C
M_A
L
0
$\frac{1}{4}L_B$
$2\ M_B$
$\frac{3}{4}L_B$
L_b
$\frac{1}{2}\frac{L}{B}$
*** ABS. VALUE ONLY

BEAM RESPONSE CURVE

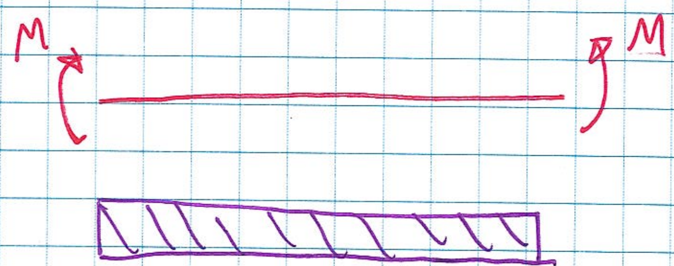


MORE ON C_b

- only app. when $L_b > L_p$
- "continuously" braced $\Rightarrow$ $L_b = 0$
- "fully braced" $\Rightarrow$ $L_b < L_p$

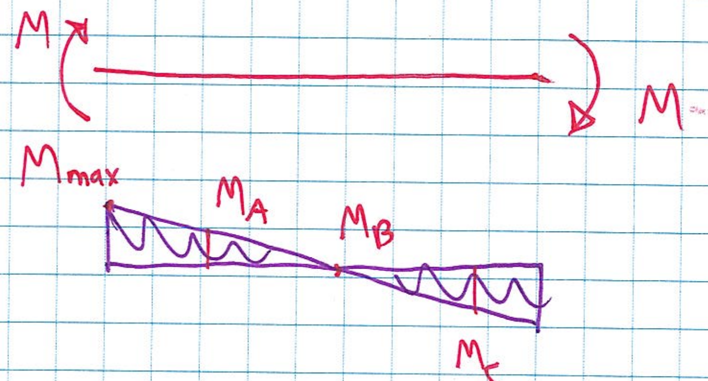
$$\left.\begin{matrix} L_b = 0 \\ L_b < L_p \end{matrix}\right\} \phi_b M_p = 0.9 f_y Z_x = \phi_b M_n$$

CASE 1



$$\left.\begin{matrix} M_{max} = M \\ M_A = M_B = M_C = M \end{matrix}\right\} \underline{\underline{C_b = 1.0}}$$

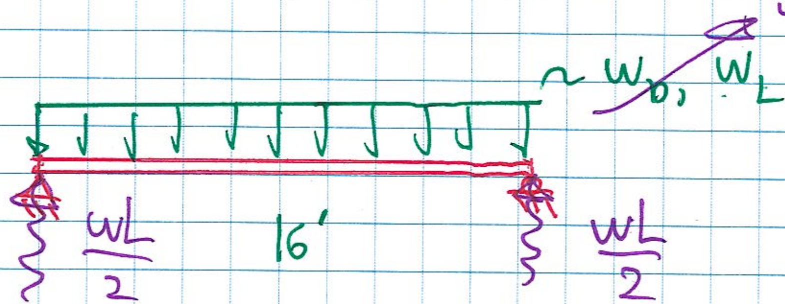
CASE 2



$C_b = 2.27$ "best case"

EXAMPLE

GIVEN: A simply supported beam W24X103, A992 Steel, Continuously braced, and subjected to service loads w_D, w_L and $W_L = 2W_D$



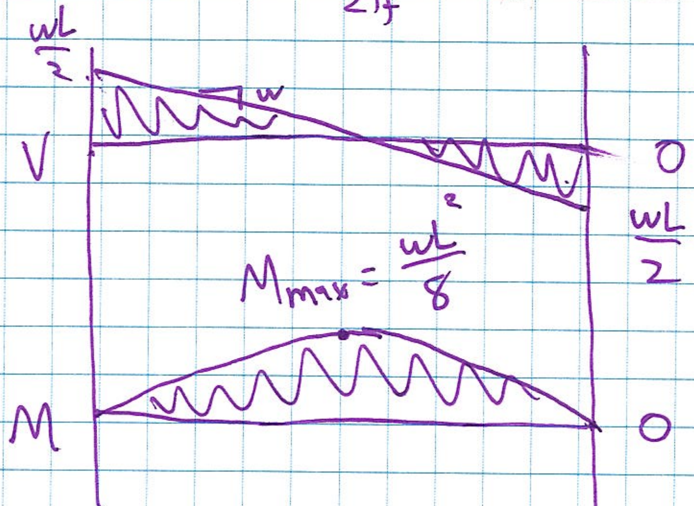
$L_b = 0$

$\phi_b M_n = \phi_b M_p$

REQ'D: DETERMINE THE MAX SERVICE LOAD THE BM. CAN CARRY

SOLN: W24X103: $Z_x = 280 \text{ in}^3$

$\frac{b_f}{2t_f} = 4.59$; $\frac{h}{t_w} = 39.2$



$4.59 \le \frac{65}{\sqrt{F_y}} = 9.2$ ✓

$39.2 \le \frac{640}{\sqrt{F_y}} = 90.5$ ✓

compact — no stability issues w/ FLB or WLB

$$\phi_b M_n = \phi_b M_p = \phi_b f_y Z_x = 0.9\,(50^{ksi})(280\ in^3)\left(\tfrac{1}{12}\right) = \underline{1050}\ ^{k\text{-}ft}$$

$$M_{max} \leq \phi_b M_n$$

$$\frac{WL^2}{8} = 1050^{k\text{-}ft} \Rightarrow W_u = \frac{(1050^{k\text{-}ft})(8)}{(16\ ft)^2} = 32.8\ ^{k}/_{ft}$$

$$W_u = 1.2\,W_D + 1.6\,W_L$$

$$32.8\ ^{k}/_{ft} = 1.2\,(W_D) + 1.6\,(2W_D) \Rightarrow W_D = 7.43\ ^{k}/_{ft}$$

$$W_L = 14.86\ ^{k}/_{ft}$$

$$W_{service} = W_D + W_L = \underline{22.3}\ ^{k}/_{ft}$$