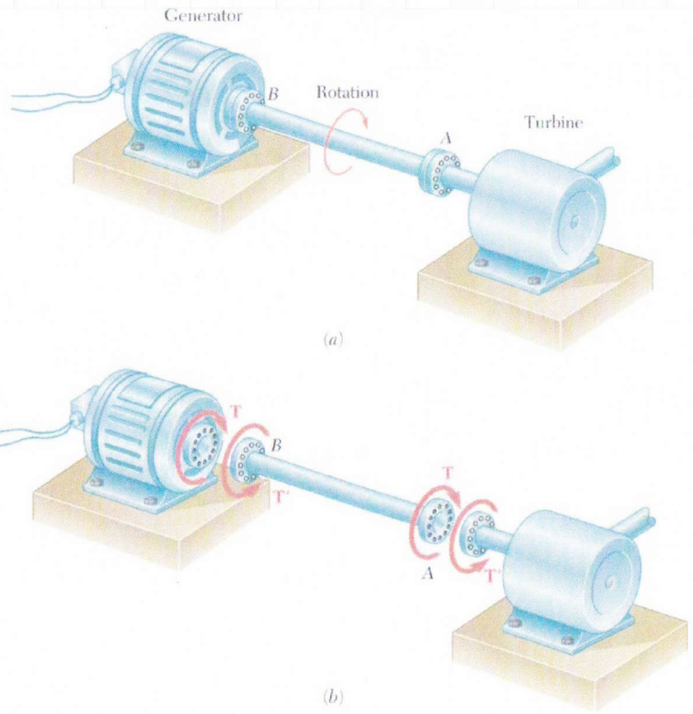
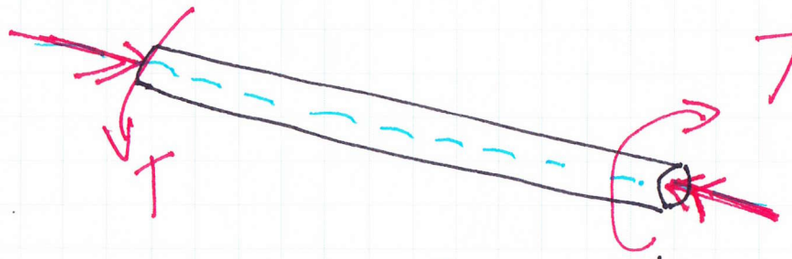
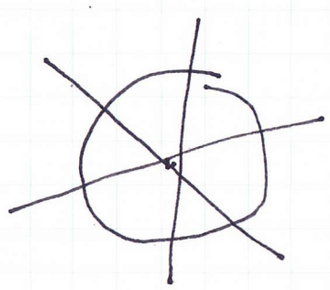
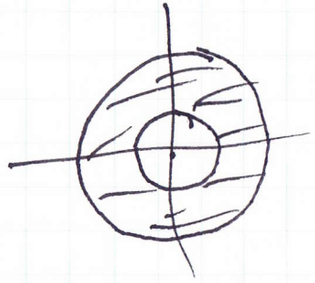
Torsion of Circular Shafts

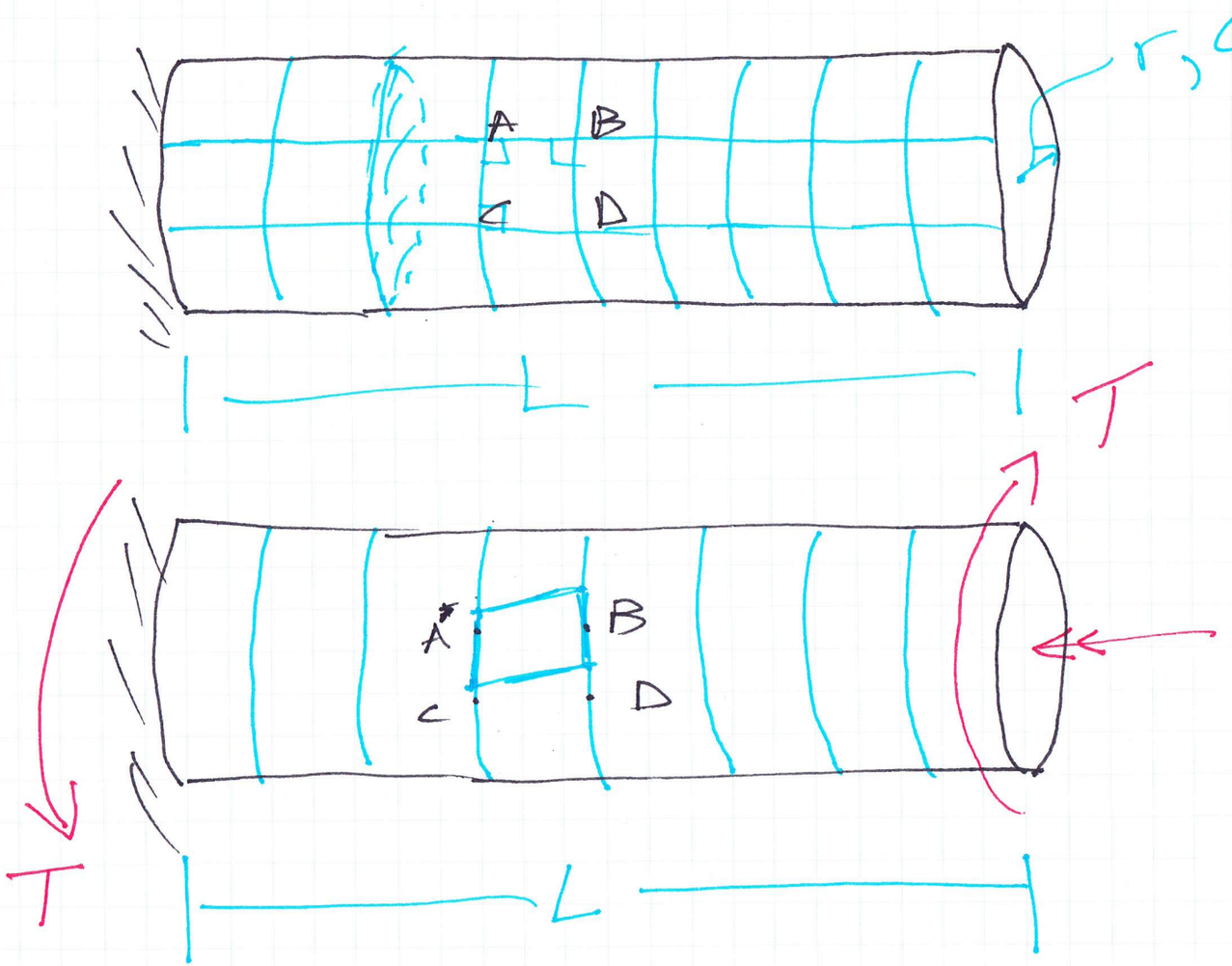




Axially Symmetric Cross-Sections

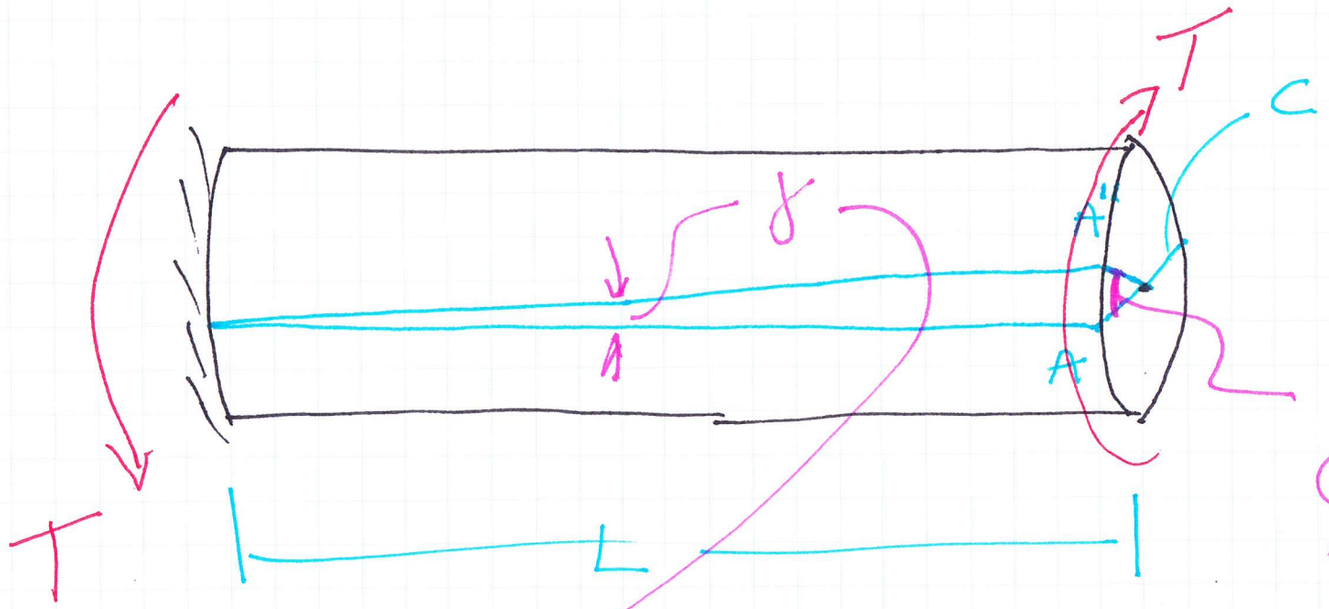






Plane Sections Remain Plane

Torque Causes Shearing Stress in the Shaft.



Plane Sections Remain Plane

Straight lines Remain Straight

ϕ — Angle of Twist

γ — Shearing Strain

AA' — approximately a straight line.

$$AA' = c\phi$$

$$AA' = \gamma L$$

$$c\phi = \gamma L$$

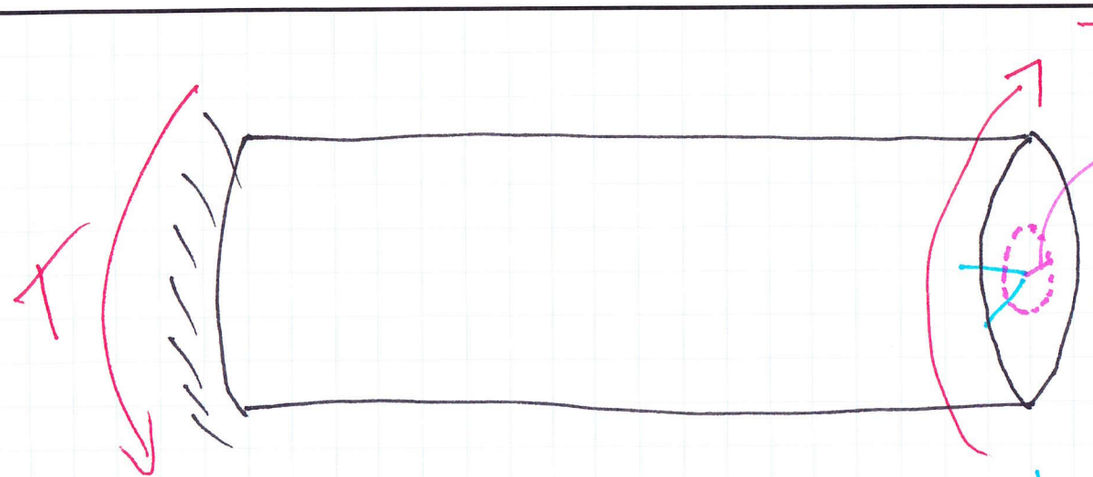
$$\boxed{\gamma = \frac{c\phi}{L}}$$

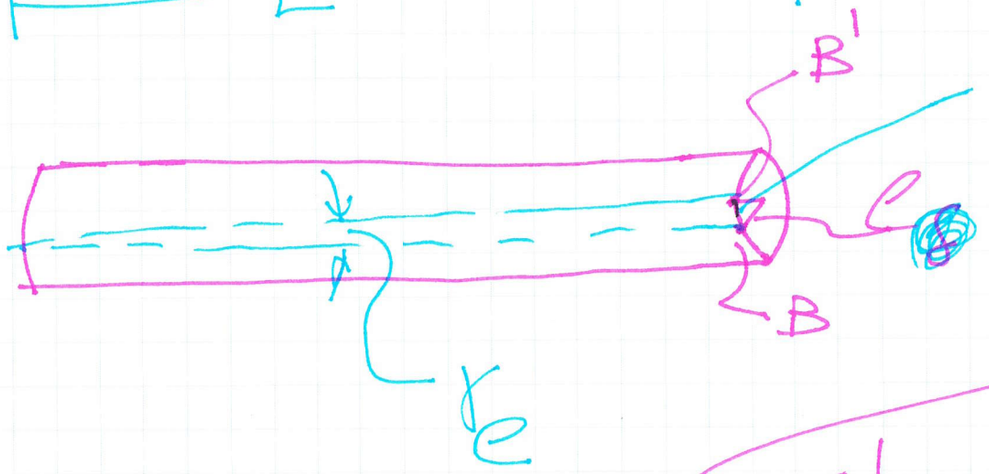
radius

Angle of twist radians

shearing strain radians

Length of the shaft





$$e\phi = \gamma_e L$$

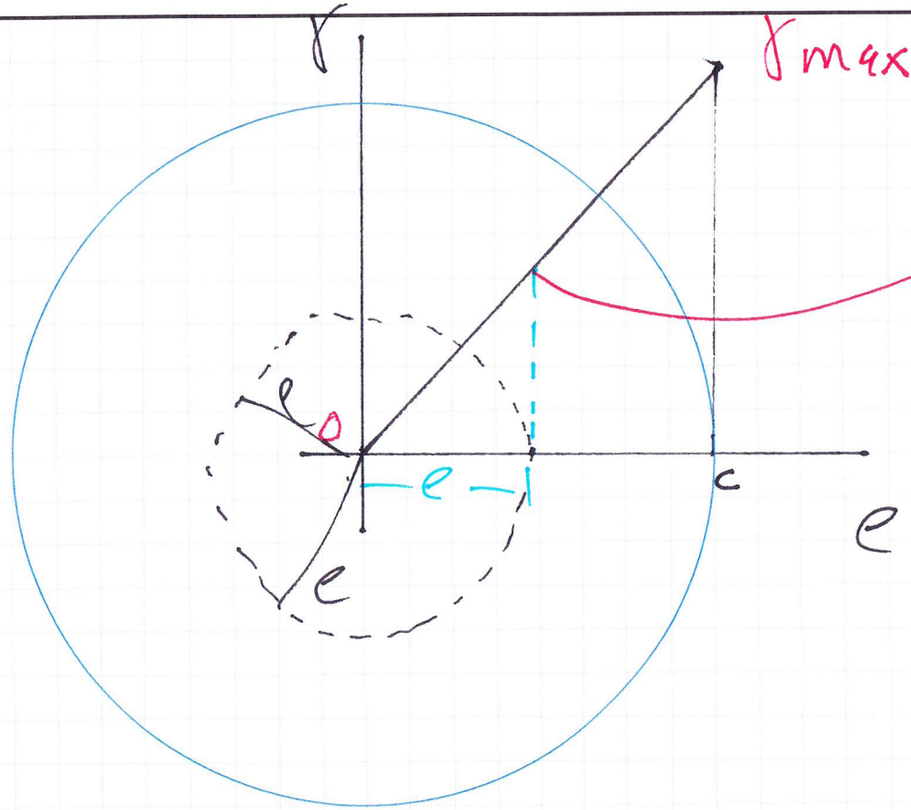
$$\gamma_e = \frac{e\phi}{L}$$

Angle of twist

Length

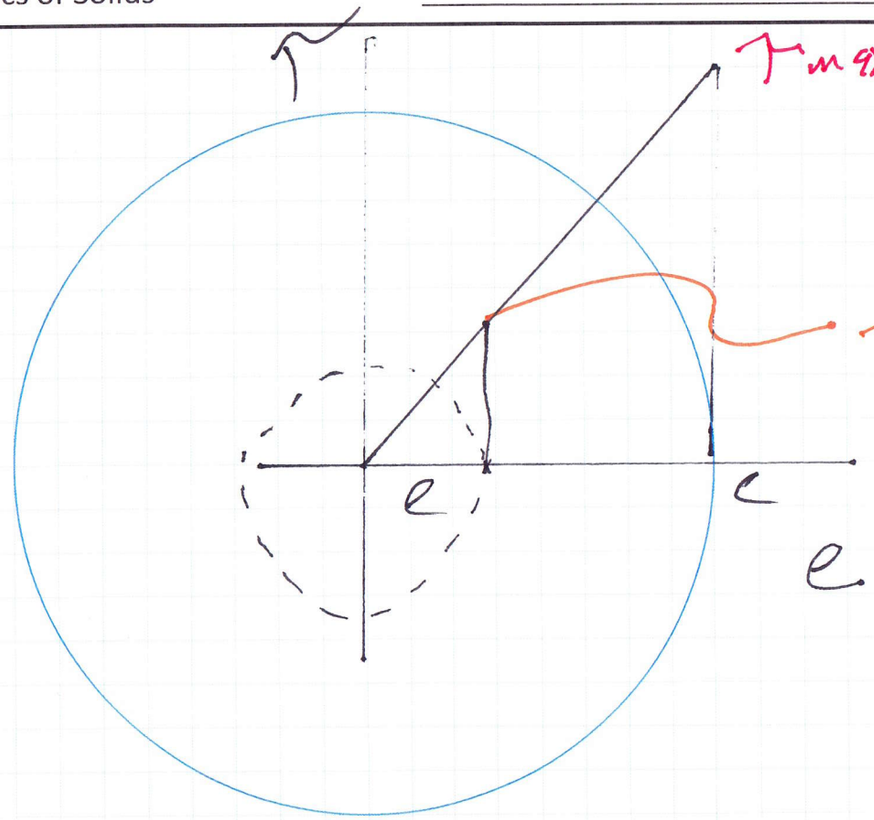
any radius measure from the center of the circular cross section

Shearing strain at any distance e



$$\gamma = \frac{e\phi}{L}$$

$$\gamma = \frac{e}{c}\gamma_{max}$$

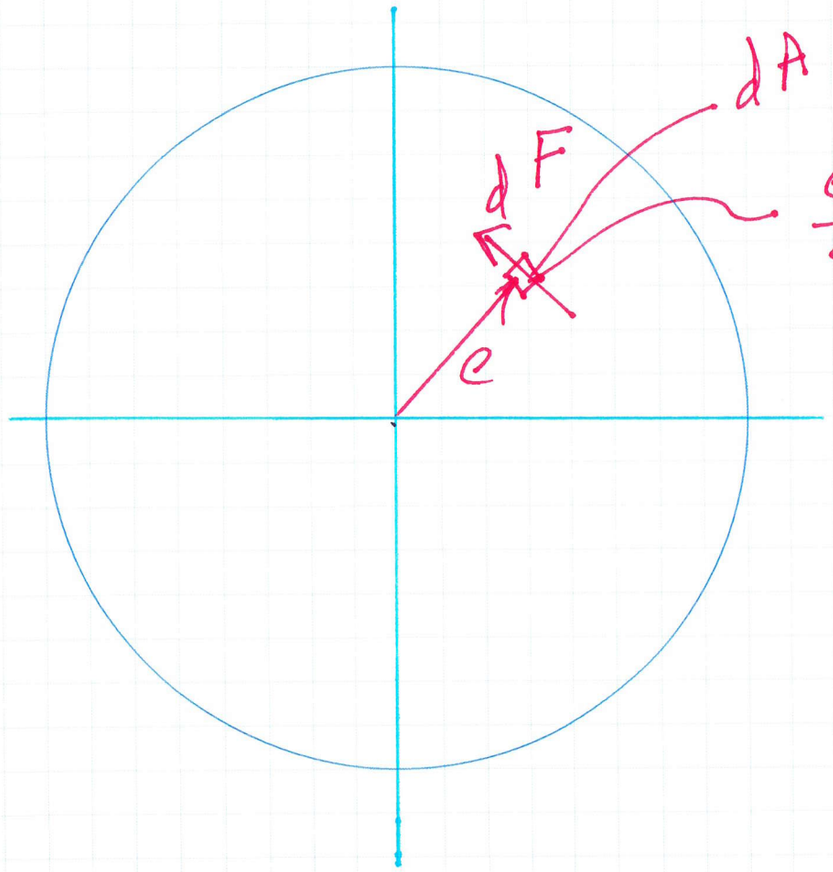


$$\tau = G\gamma$$

$$\tau = G\frac{\rho}{c}\gamma_{max}$$

$$\tau = G\frac{\rho}{c}\gamma_{max}$$

$$\boxed{\tau = \frac{\rho}{c}\tau_{max}}$$



$$dF = \tau\, dA$$

$$dF = \frac{\rho}{c}\tau_{max}\, dA$$

$$dT = \rho\, dF$$

$$dT = \rho\, \frac{\rho}{c}\tau_{max}\, dA$$

$$T = \int_{Area} \frac{\tau_{max}}{c}\rho^2\, dA$$

$$T = \frac{\tau_{max}}{c} \int_{Area} \rho^2 \, dA$$

J

$$T = \frac{\tau_{max}}{c} J$$

Polar Moment of Inertia

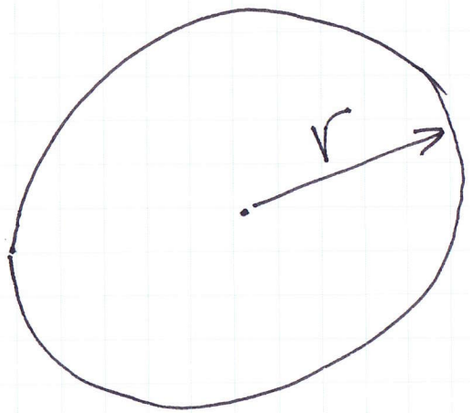
$$\boxed{\tau_{max} = \frac{Tc}{J}}$$

c = distance from center

$$\tau = \frac{T\rho}{J}$$

τ = Shearing stress at a point ρ from the center

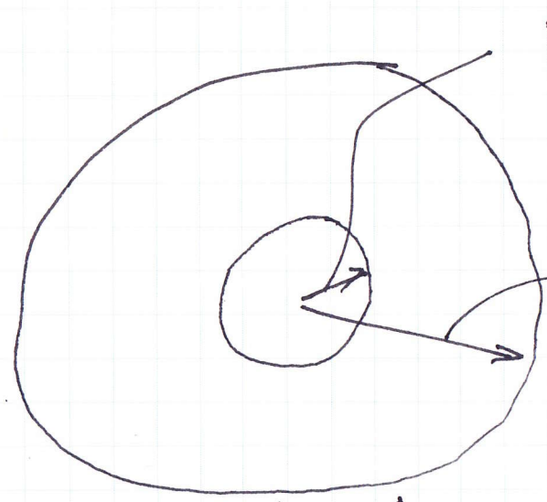
Solid Shaft



in^4
m^4

$$J = \frac{\pi}{2} r^4$$

Hollow Shaft



$$J = \frac{\pi}{2}\left(r_{outer}^4 - r_{inner}^4\right)$$

$$a^4 - b^4 \neq (a-b)^4$$