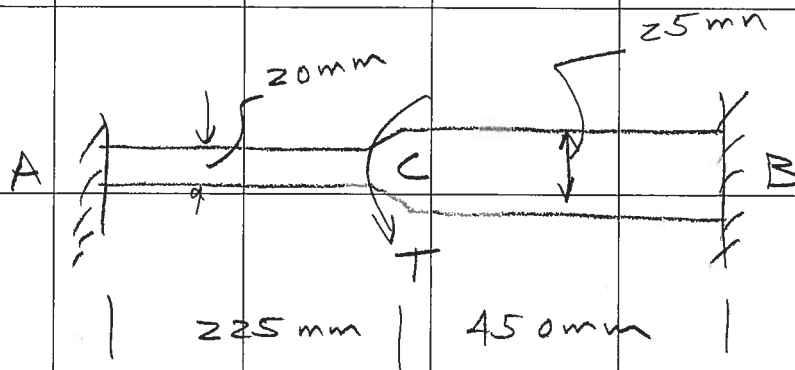


A stepped shaft ACB having solid circular cross-sections with two different diameters is fixed at both ends as shown. What is the maximum torque, T , that may be applied at point C if the maximum shearing stress in the shaft is not to exceed 45 MPa. Assume that $G = 80 \text{ GPa}$.



First Calculate I_p for shafts

$$I_{pAC} = \frac{\pi}{2} \left(\frac{.020}{2} \right)^4 \quad (1)$$

$$I_{pAC} = 1.571 \times 10^{-8} \text{ m}^4 \quad (2)$$

$$I_{pCB} = \frac{\pi}{2} \left(\frac{.025}{2} \right)^4 \quad (3)$$

$$I_{pCB} = 3.835 \times 10^{-8} \text{ m}^4 \quad (4)$$

Compatibility relationships suggest that

$$|\phi_{AC}| = |\phi_{CB}| \quad (5)$$

so

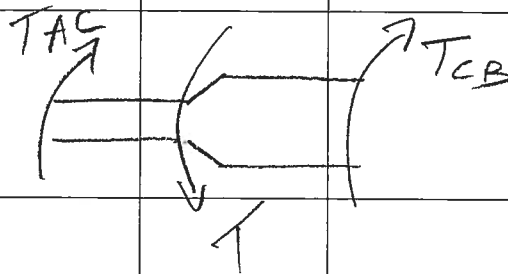
$$\frac{T_{AC} L_{AC}}{I_{PAC} G} = \frac{T_{CB} L_{CB}}{I_{PCB} G} \quad (6)$$

$$\Rightarrow \frac{T_{AC} (1.225 \text{ m})}{(1.571 \times 10^{-9}) (80 \times 10^9)} = \frac{T_{CB} (1.450 \text{ m})}{(3.835 \times 10^{-8} \text{ m}^4) (80 \times 10^9 \frac{\text{N}}{\text{m}^2})}$$

$$T_{AC} = .8193 T_{CB} \quad (8)$$

$$\text{or } T_{CB} = 1.221 T_{AC} \quad (9a)$$

Consider a F.B.D of central segment of the shaft that includes point C.



$$\Rightarrow \sum M_{x\text{-axis}} = 0$$

$$T - T_{AC} - T_{CB} = 0 \quad (9)$$

$$T = T_{AC} + T_{CB} \quad (10)$$

Assume Segment AC controls.

$$\tau_{AC} = \frac{T_{AC} r_{AC}}{I_{PAC}} \quad (11)$$

$$45 \times 10^6 \frac{N}{m^2} = \frac{T_{AC} (.010 m)}{1.571 \times 10^{-8} m^4} \quad (12)$$

$$T_{AC} = 70.7 \text{ N-m} \quad (13)$$

Then using (8a) and (10) we get

$$T = T_{AC} + T_{CB} \quad (14)$$

$$T = 70.7 + 1.221(70.7) \quad (15)$$

$$T = 157.0 \text{ N-m} \quad (16)$$

This might be max torque

Assume Segment CB controls

$$\tau_{CB} = \frac{T_{CB} r_{CB}}{I_{PCB}} \quad (17)$$

$$45 \times 10^6 \frac{N}{m^2} = \frac{T_{CB} (.0125 m)}{3.835 \times 10^{-8} m^4} \quad (18)$$

$$T_{CB} = 138.1 \text{ N-m} \quad (19)$$

Then using (8) and (10)

$$T = T_{AC} + T_{CB} \quad (20)$$

$$T = (.8193)(138.1) + 138.1 \quad (21)$$

$$T = 251.2 \text{ N-m} \quad (22)$$

This might be maximum Torque

Because $251.2 \text{ N-m} > 157.0 \text{ N-m}$ (23)
then segment AC controls

Thus, the maximum allowable
torque, T , is 157.0 N-m