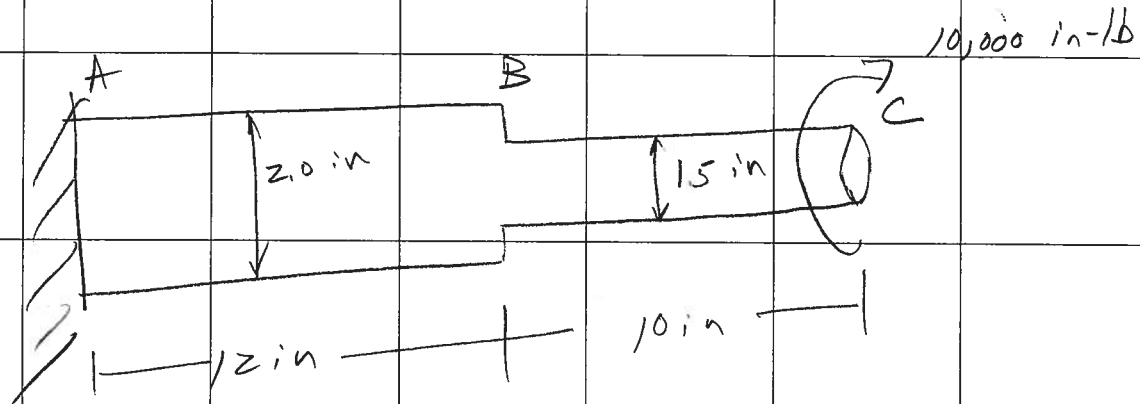
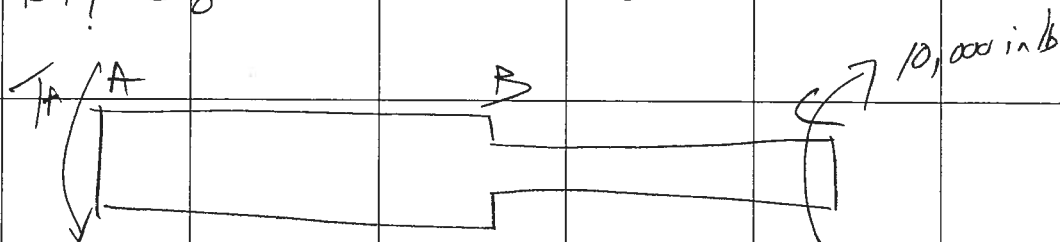


The shaft shown is made of steel ($G = 11 \times 10^6 \frac{\text{lb}}{\text{in}^2}$). Calculate the total strain energy in the shaft and the angle of twist at point C using energy methods.



Apply equations of equilibrium



$$\sum T = 0$$

$$T_A - 10,000 = 0$$

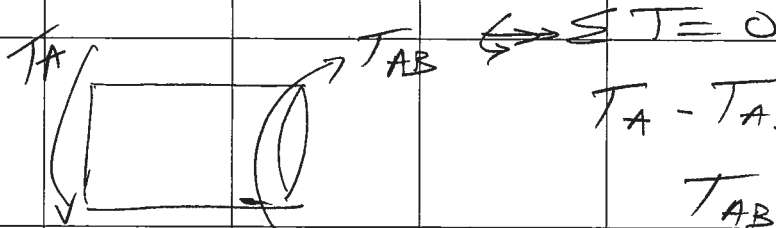
(1)

$$T_A = 10,000 \text{ in-lb}$$

as shown

(2)

Section AB



$$T_A - T_{AB} = 0 \quad (3)$$

$$T_{AB} = 10,000 \text{ in-lb} \quad (4)$$

$$I_{PAB} = \frac{\pi}{2} \left[\left(\frac{2.0}{2} \right)^4 - \left(\frac{0}{2} \right)^4 \right] \quad (5)$$

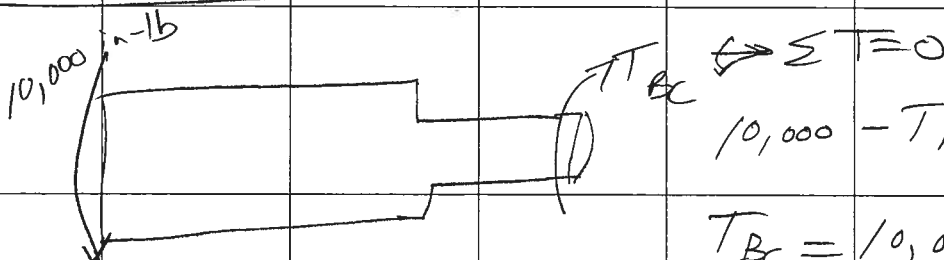
$$I_{PAB} = 1.571 \text{ in}^4 \quad (6)$$

$$U_{AB} = \frac{T_{AB}^2 L_{AB}}{2 G_{AB} I_{PAB}} \quad (7)$$

$$U_{AB} = \frac{(10,000 \text{ in-lb})^2 (12)}{2 (1.571 \text{ in}^4) (11 \times 10^6 \frac{\text{lb}}{\text{in}^2})} \quad (8)$$

$$U_{AB} = 34.72 \text{ in-lb} \quad (9)$$

Section BC



$$10,000 - T_{BC} = 0 \quad (10)$$

$$T_{BC} = 10,000 \text{ in-lb} \quad (11)$$

$$I_{PBC} = \frac{\pi}{2} \left[\left(\frac{1.5}{2} \right)^4 - \left(\frac{0}{2} \right)^4 \right] \quad (12)$$

$$I_{PBC} = .497 \text{ in}^4 \quad (13)$$

$$U_{BC} = \frac{T_{BC}^2 L_{BC}}{2 G_{BC} I_{PBC}} \quad (14)$$

$$U_{BC} = \frac{(10,000 \text{ in-lb})^2 (10 \text{ in})}{2 (11 \times 10^6 \frac{\text{lb}}{\text{in}^2}) (.497 \text{ in}^4)} \quad (15)$$

$$U_{BC} = 91.46 \text{ in-lb} \quad (16)$$

The total strain energy is

$$U_{Total} = U_{AB} + U_{BC} \quad (17)$$

$$U_{Total} = 34.72 + 91.46 \quad (18)$$

$$U_{Total} = 126.2 \text{ in-lb} \quad (19)$$

The external work is

$$W = \frac{1}{2} (10,000 \text{ in-lb}) \phi_{AC} \quad (20)$$

Then

$$W = U \quad (21)$$

$$\frac{1}{2} (10,000 \text{ in-lb}) \phi_{AC} = 126.2 \text{ in-lb} \quad (22)$$

$$\phi_{AC} = .0252 \text{ radians} \quad (23)$$

Check using Conventional Approach

$$\phi_{AC} = \phi_{AB} + \phi_{BC} \quad 24$$

$$\phi_{AC} = \frac{(10,000)(12)}{(1.571)(11 \times 10^6)} + \frac{(10,000)(10)}{(.4971)(11 \times 10^6)} \quad 25$$

$$\phi_{AC} = .0252 \text{ radians} \quad \text{OK} \quad 26$$