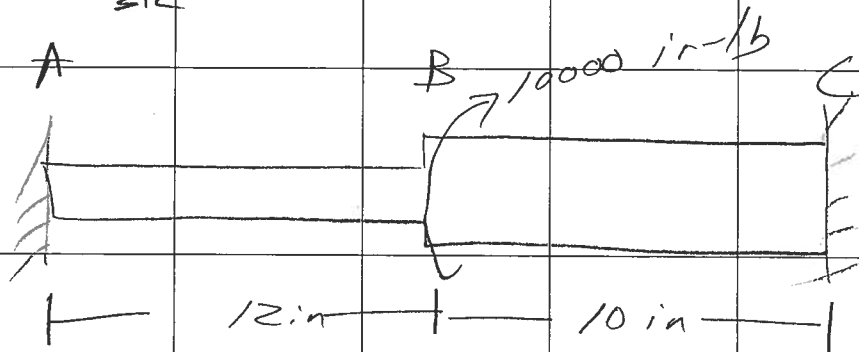
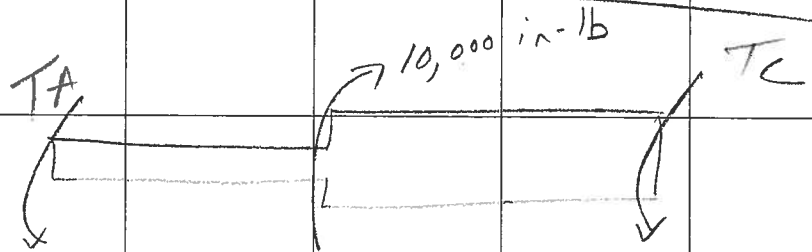


A solid steel shaft is fixed on both ends as shown. The diameter of section AB is 2 in. and the diameter of section BC is 3 in. Calculate the support reactions and the maximum shearing stress in the shaft. Assume that $G_{STE} = 11 \times 10^6 \frac{\text{lb}}{\text{in}^2}$.



Equilibrium Relationships

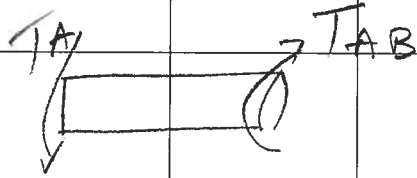


$$\sum T = 0$$

$$T_A + T_C - 10,000 \text{ in-lb} = 0 \quad (1)$$

$$T_A + T_C = 10,000 \text{ in-lb} \quad (2)$$

Examine Section AB



$$I_{PAB} = \frac{\pi}{2} (k_o^4 - k_i^4) \quad (3)$$

$$I_{PAB} = \frac{\pi}{2} ((1\text{ in})^4 - 0^4) \quad (4)$$

$$I_{PAB} = 1.571 \text{ in}^4 \quad (5)$$

$$\sum T = 0$$

$$T_A - T_{AB} = 0 \quad (6)$$

$$T_{AB} = T_A \quad (7)$$

$$\tau_{AB} = \frac{T_{AB} r_{AB}}{I_{PAB}} \quad (8)$$

$$\phi_{AB} = \frac{T_{AB} L_{AB}}{I_{PAB} G_{AB}} \quad (11)$$

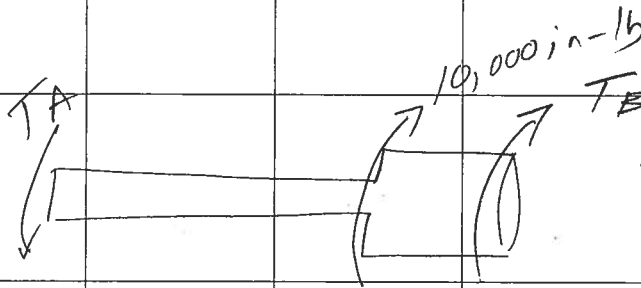
$$\tau_{AB} = \frac{T_A (1\text{ in})}{1.571 \text{ in}^4} \quad (9)$$

$$\phi_{AB} = \frac{T_A (12\text{ in})}{(1.571 \text{ in}^4) G_{STL}} \quad (12)$$

$$\tau_{AB} = .6365 T_A \quad (10)$$

$$\phi_{AB} = 7.638 \frac{T_A}{G_{STL}} \quad (13)$$

Examine Section BC



$$I_{PBC} = \frac{\pi}{2} (r_o^4 - r_i^4) \quad (14)$$

$$I_{PBC} = \frac{\pi}{2} \left(\left(\frac{3}{2} \right)^4 - (0)^4 \right) \quad (15)$$

$$I_{PBC} = 7.952 \text{ in}^4 \quad (16)$$

$$\sum T = 0$$

$$T_A - 10,000 - T_{BC} = 0 \quad (17)$$

$$T_{BC} = T_A - 10,000 \quad (18)$$

$$\tau_{BC} = \frac{T_{BC} r_{BC}}{I_{PBC}}$$

$$\phi_{BC} = \frac{T_{BC} L_{BC}}{I_{PBC} G_{st}} \quad (21)$$

$$\tau_{BC} = \frac{(T_A - 10,000)(1.75)}{7.952}$$

$$\phi_{BC} = \frac{(T_A - 10,000)(10 \text{ in})}{(7.952 \text{ in}^4) G_{st}} \quad (22)$$

$$\tau_{BC} = .220 T_A - 2201 \quad (23)$$

$$\phi_{BC} = 1.258 \frac{T_A}{G_{st}} - 12,580 \frac{1}{G_{st}} \quad (24)$$

Compatibility

$$\phi_{AB} + \phi_{BC} = \phi_{AC} = 0 \quad (25)$$

$$7.638 \frac{T_A}{GJL} + 1.258 \frac{T_A}{GJL} - 12580 \frac{1}{GJL} = 0 \quad (26)$$

$$8.896 T_A = 12580 \quad (27)$$

$$T_A = 1414.1 \text{ in-lb} \quad (28)$$

Then plugging back into equation (2)

$$T_A + T_C = 10000 \text{ in-lb} \quad (29)$$

$$1414.1 + T_C = 10000 \quad (30)$$

$$T_C = 8585.9 \text{ in-lb} \quad (31)$$

Check stresses

From equation (21)

$$T_{BC} = .220 T_A - 2201 \quad (32)$$

$$T_{BC} = .220 (1414.1) - 2201 \quad (33)$$

$$\tau_{BC} = -1890 \frac{\text{lb}}{\text{in}^2}$$

(31)

From equation (10)

$$\tau_{AB} = .6365 T_A$$

(35)

$$\tau_{AB} = (.6365)(1414.1)$$

(36)

$$\tau_{AB} = 900 \text{ psi}$$

(37)

Materials do not care whether
shearing stresses are + or - ,
All that matters is magnitude.

$$\text{because } |-1890| > |900| \quad 38$$

the maximum shearing stress
occurs in section BC and
is equal to 1890 psi