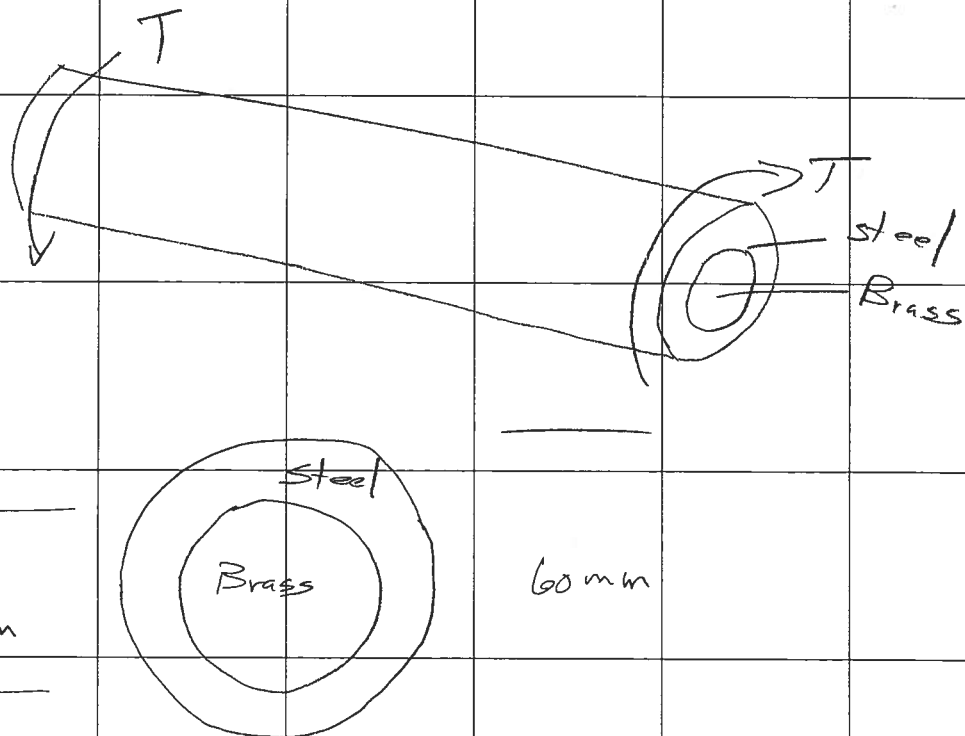


The composite shaft shown is manufactured by shrink-fitting a steel sleeve over a brass core so that the two parts act as a single bar in torsion. The outer diameter of the steel is 60 mm and the diameter of the brass is 50 mm. Assume $G_{st} = 80 \text{ GPa}$ and $G_{br} = 36 \text{ GPa}$. Calculate the maximum allowable torque that can be applied to the shaft if the allowable shearing stress for brass is 48 MPa and the allowable shearing stress for steel is 80 MPa.



11)

$$(N)$$

$$(\approx)$$

(4)

(5)

(5a)

$\hookrightarrow \sum T = 0$

(6)

(7)

Compatibility relationship

Because the hollow steel is bonded with the brass, the angle of twist of the steel is equal to the angle of twist of the brass

$$\phi_{st} = \phi_{br} \quad (8)$$

$$\frac{T_{st} L_{st}}{I_{p_{st}} G_{st}} = \frac{T_{br} L_{br}}{I_{p_{br}} G_{br}} \quad (9)$$

$$\frac{T_{st} \cancel{L}}{(6.588 \times 10^{-7})(80 \times 10^9)} = \frac{T_{br} \cancel{L}}{(6.136 \times 10^{-7})(36 \times 10^9)} \quad (10)$$

$$T_{st} = 2.386 T_{br} \quad (11)$$

$$T_{br} = .4191 T_{st} \quad (11a)$$

Check allowable stresses for steel

$$\tau_{allow} = \frac{T_{st} r_{st}}{I_{p_{st}}} \quad (12)$$

$$80 \times 10^6 \frac{N}{m^2} = \frac{T_{st} (.030 m)}{6.588 \times 10^{-7} m^4} \quad (13)$$

$$T_{st} = 1,757 \text{ N-m} \quad (14)$$

Then plugging (14) and (119) into (7)

$$T_{st} + .4191 T_{st} = T_{Allow} \quad (15)$$

$$1757 \text{ N-m} + .4191 (1757 \text{ N-m}) = T_{Allow} \quad (16)$$

$$T_{Allow} = 2493 \text{ N-m} \quad (17)$$

Based on steel

Check allowable stresses for brass

$$\tau_{Allow} = \frac{T_{Br} r_{Br}}{J_{Br}} \quad (18)$$

$$48 \times 10^6 \frac{\text{N}}{\text{m}^2} = \frac{T_{Br} (.025 \text{ m})}{(6.136 \times 10^{-7} \text{ m}^4)} \quad (19)$$

$$T_{Br} = 1,178 \text{ N-m} \quad (20)$$

Then plugging (20) and (11) into (7)

$$2.386 T_{Br} + T_{Br} = T_{Allow} \quad (21)$$

$$2.386 (1178) + 1178 = T_{Allow} \quad (22)$$

$$T_{Allow} = 3989 \text{ N-m} \quad (23)$$

Based on brass

Because

$$2493 \text{ N-m} < 3989 \text{ N-m}$$

the allowable steel stress controls

Thus the allowable torque on the
shaft is

$$T_{\text{Allow}} = 2493 \text{ N-m}$$

(24)