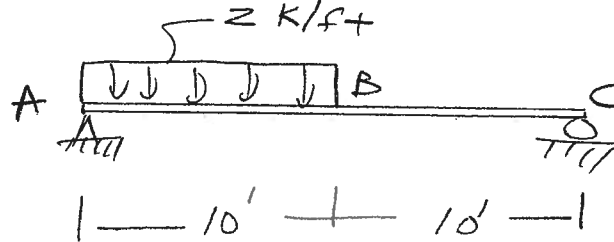
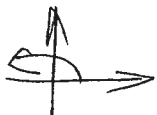
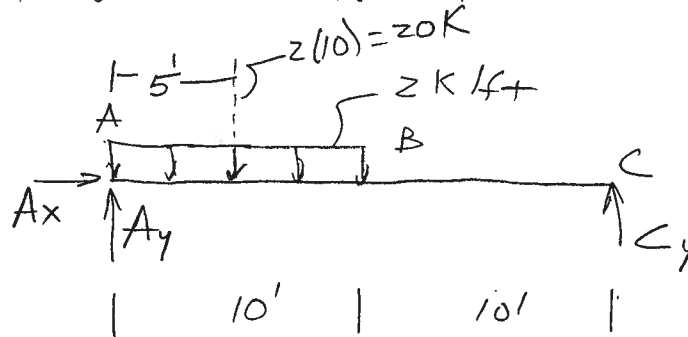


A beam is loaded as shown. Use double integration and singularity functions to determine the equation of the elastic curve. Assume that EI is constant.



First we calculate the external reactions



$$\sum F_x = 0$$

$$(1) \quad A_x = 0$$

$$\sum M_C = 0$$

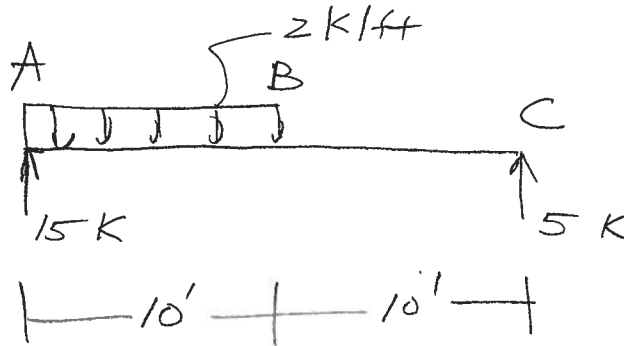
$$(2) \quad -A_y(20) + 20(15) = 0$$

$$(3) \quad A_y = \underline{15 \text{ K}}$$

$$\sum F_y = 0$$

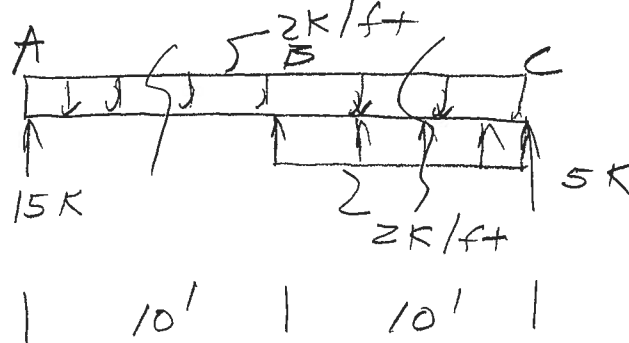
$$(4) \quad A_y - 20 + C_y = 0$$

$$(5) \quad \underline{C_y = 5}$$

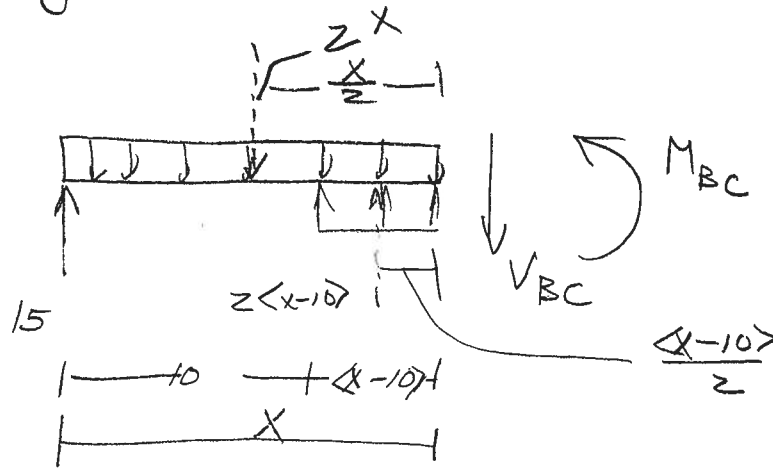


When singularity functions are applied to a uniform load it is possible to start the load, but it is not possible to stop the load. Therefore, once a load enters the equation, it remains for the full length of the beam.

To get around this problem we apply an equivalent load as shown



We choose the section furthest to the right



$$\sum M_{cut} = 0$$

$$(6) \quad M_{BC} + z \times \left(\frac{X}{2}\right) - z \langle X-10 \rangle \frac{\langle X-10 \rangle}{2} - 15X = 0$$

$$(7) \quad M_{BC} = \langle X-10 \rangle^2 - X^2 + 15X$$

Next we write the differential equation and integrate twice

$$(8) \quad EI \frac{d^2y}{dx^2} = \langle X-10 \rangle^2 - X^2 + 15X$$

$$(9) \quad EI \frac{dy}{dx} = \frac{1}{3} \langle X-10 \rangle^3 - \frac{1}{3} X^3 + \frac{15}{2} X^2 + C_1$$

$$(10) \quad EI y = \frac{1}{12} \langle X-10 \rangle^4 - \frac{1}{12} X^4 + \frac{15}{6} X^3 + C_1 X + C_2$$

Next we apply the boundary conditions

@ $x=0$, $y=0$ for equation (10)

$$(11) \quad 0 = \frac{1}{12} \langle 0-10 \rangle^4 - \frac{1}{12} (0)^4 + \frac{15}{6} (0)^3 + C_1 (0) + C_2$$

$$(12) \quad \therefore C_2 = 0$$

@ $x=20$, $y=0$ for equation (10)

$$(13) \quad 0 = \frac{1}{12} \langle 20-10 \rangle^4 - \frac{1}{12} (20)^4 + \frac{15}{6} (20)^3 + C_1 (20)$$

$$(14) \quad 0 = 833.33 - 13,333.33 + 20,000 + C_1 (20)$$

$$(15) \quad 20 C_1 = -7500.00$$

$$\underline{C_1 = -375}$$

Finally we write the equations for the slope and deflection

$$(16) \quad \frac{dy}{dx} = \frac{1}{EI} \left[\frac{1}{3} \langle x-10 \rangle^3 - \frac{1}{3} x^3 + \frac{15}{2} x^2 - 375x \right]$$

$$(17) \quad y = \frac{1}{EI} \left[\frac{1}{12} \langle x-10 \rangle^4 - \frac{1}{12} x^4 + \frac{15}{6} x^3 - 375x \right]$$