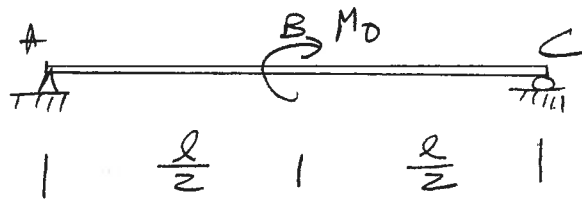
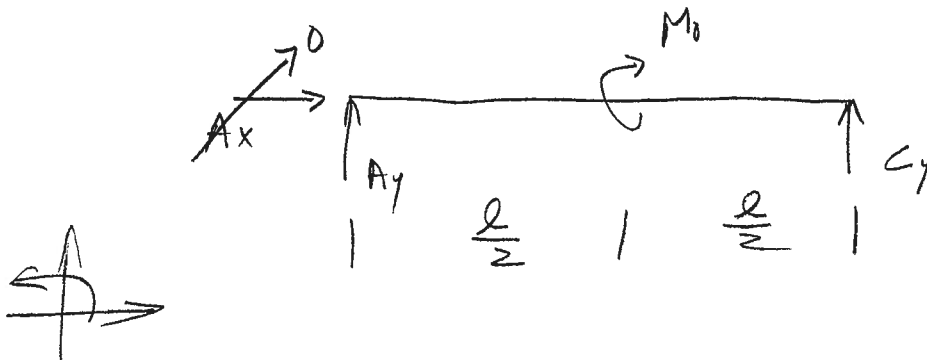


A beam is loaded as shown.. Use double integration and singularity functions to determine the equation of the elastic curve. Assume that EI is constant.



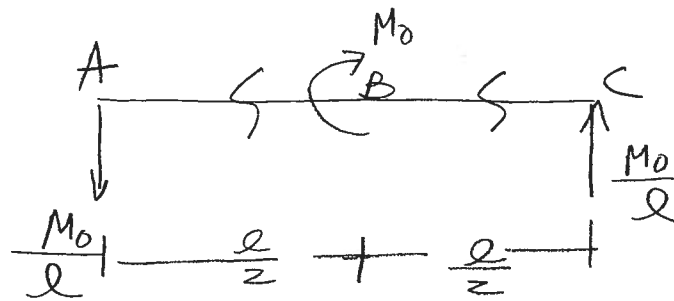
The 1st step is to calculate the external reactions



$$\begin{aligned} \sum F_x = 0 & \quad \sum M_C = 0 \\ A_x = 0 & \quad -M_0 - A_y l = 0 \quad (2) \\ & \quad A_y = -\frac{M_0}{l} \quad (3) \end{aligned}$$

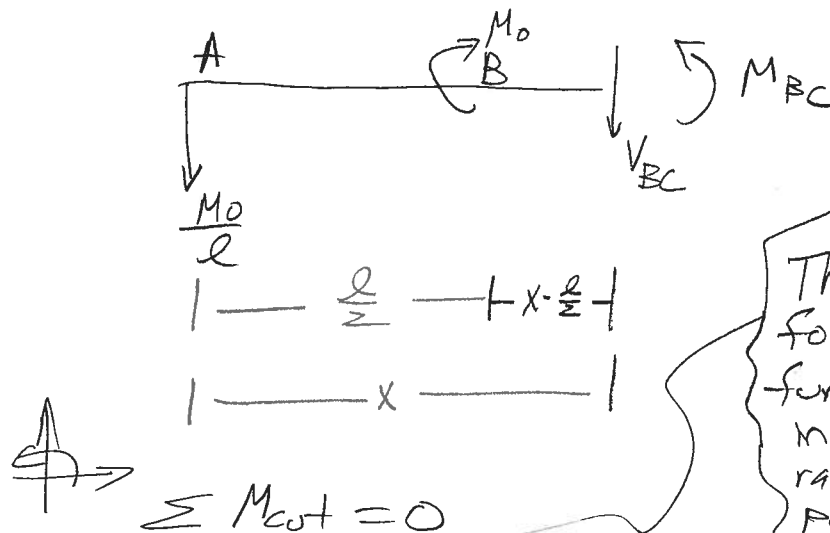
$$\begin{aligned} \sum F_y = 0 & \quad A_y + C_y = 0 \\ & \quad A_y + C_y = 0 \end{aligned}$$

$$\begin{aligned} C_y &= \frac{M_0}{l} \end{aligned}$$



If conventional moment equations were to be written, it would require separate equations for sections AB and BC.

With singularity functions we choose the section furthest to the right, BC



This is the proper form of the singularity function for an applied moment. Anything raised to the zero power is 1.0

$$(6) \quad M_{BC} - M_0 \left\langle x - \frac{l}{2} \right\rangle^0 + \frac{M_0}{l} x = 0$$

$$(7) \quad M_{BC} = -\frac{M_0}{l} x + M_0 \left\langle x - \frac{l}{2} \right\rangle^0$$

Next we write the differential equation and integrate twice.

$$(8) \quad EI \frac{d^2 y}{dx^2} = -\frac{M_0}{2} X + M_0 \left\langle X - \frac{L}{2} \right\rangle^0$$

$$(9) \quad EI \frac{dy}{dx} = -\frac{M_0}{2L} X^2 + M_0 \left\langle X - \frac{L}{2} \right\rangle^1 + C_1$$

$$(10) \quad EI y = -\frac{M_0}{6L} X^3 + \frac{M_0}{2} \left\langle X - \frac{L}{2} \right\rangle^2 + C_1 X + C_2$$

Next we apply boundary conditions

@ $X=0$, $y=0$ for equation (10)

$$(11) \quad 0 = -\frac{M_0}{6L} (0)^3 + \frac{M_0}{2} \left\langle 0 - \frac{L}{2} \right\rangle^2 + C_1(0) + C_2$$

$$(12) \quad \therefore C_2 = 0$$

This is always the case when $y=0$ @ $X=0$

@ $X=L$, $y=0$ for equation (10)

$$(13) \quad 0 = -\frac{M_0}{6L} (L)^3 + \frac{M_0}{2} \left\langle L - \frac{L}{2} \right\rangle^2 + C_1 L$$

$$(14) \quad 0 = -\frac{M_0 L^2}{6} + \frac{M_0}{2} \left(\frac{L^2}{4} \right) + C_1 L$$

$$(15) \quad 0 = -\frac{M_0 L^2}{6} + \frac{M_0 L^2}{8} + C_1 L$$

$$(16) \quad C_1 l = \frac{M_0 l^2}{24}$$

$$(17) \quad C_1 = \frac{M_0 l}{24}$$

Finally we write the equations for the slope and deflection.

$$(18) \quad \frac{dy}{dx} = \frac{1}{EI} \left[-\frac{M_0}{24} x^2 + M_0 \left\langle x - \frac{l}{2} \right\rangle' + \frac{M_0 l}{24} \right]$$

$$(19) \quad y = \frac{1}{EI} \left[-\frac{M_0}{64} x^3 + \frac{M_0}{2} \left\langle x - \frac{l}{2} \right\rangle^2 + \frac{M_0 l}{24} x \right]$$