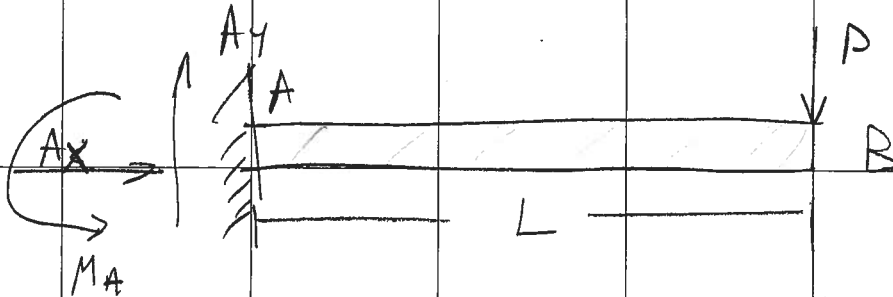
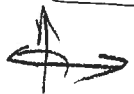


Calculate the equation of the elastic curve for the cantilever beam loaded as shown. Assume EI is constant.



First calculate the reactions for the beam



$$\sum M_A = 0$$

(1)

$$M_A - PL = 0$$

(2)

$$M_A = PL$$

$$\sum F_y = 0$$

(3)

$$A_y - P = 0$$

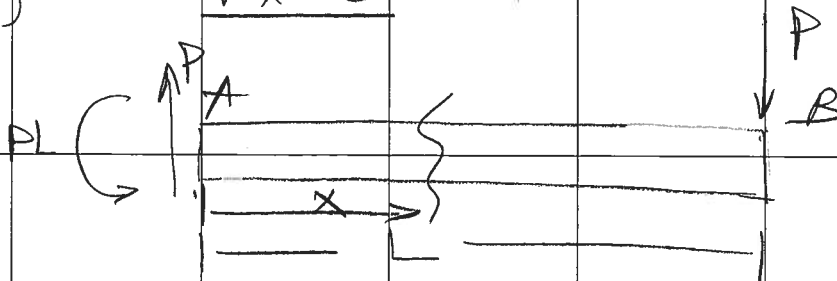
(4)

$$A_y = P$$

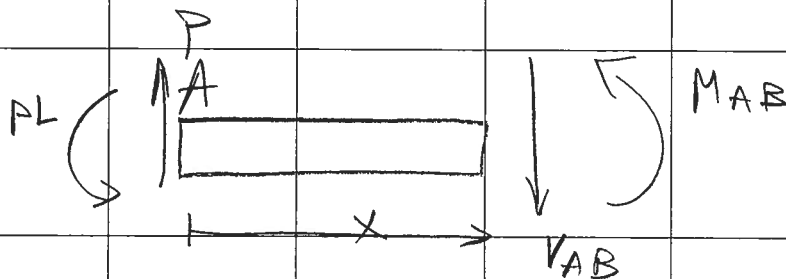
$$\sum F_x = 0$$

(5)

$$A_x = 0$$



Write the moment equation for section AB of the beam by making a cut at an arbitrary point between A and B as follows



The directions of V_{AB} and M_{AB} are based upon the assumed sign convention

$$\sum F_y = 0$$

$$(6) \quad P - V_{AB} = 0$$

$$(7) \quad \underline{V_{AB} = +P}$$

$$\sum M_{cut} = 0$$

$$(8) \quad PL - Px + M_{AB} = 0$$

$$(9) \quad \underline{M_{AB} = Px - PL}$$

This equation is good for

$$0 \leq x \leq L$$

Write differential equation for section AB

$$(10) \quad EI \frac{d^2 y}{dx^2} = P_x - PL$$

Then Integrate once and we get

$$(11) \quad EI \frac{dy}{dx} = \frac{P}{2} x^2 - PLx + C_1$$

and Integrate again to get

$$(12) \quad EI y = \frac{P}{6} x^3 - \frac{PL}{2} x^2 + C_1 x + C_2$$

The boundary conditions for the cantilever beam are $y=0 @ x=0$ and

$$\frac{dy}{dx} = 0 @ x=0$$

Plugging the second boundary condition into equation (11) results in

$$(13) \quad 0 = \frac{P}{2} (0)^2 - PL(0) + C_1$$

$$(14) \quad \therefore \underline{C_1 = 0}$$

Then plugging the first boundary condition into equation (12) results in

$$(15) \quad 0 = \frac{P}{6} (0)^3 - \frac{PL}{2} (0)^2 + C_1(0) + C_2$$

$$(16) \quad \therefore C_2 = 0$$

The equations for the slope and deflection of the beam can then be written as follows

$$(17) \quad \frac{dy}{dx} = \frac{1}{EI} \left[\frac{P}{2} x^2 - PLx \right]$$

and

$$(18) \quad y = \frac{1}{EI} \left[\frac{P}{6} x^3 - \frac{PL}{2} x^2 \right]$$

The maximum slope occurs at point B and is found by substituting $x = L$ into equation (17) as follows

$$(19) \quad \frac{dy}{dx} \bigg|_{x=L} = \frac{1}{EI} \left[\frac{P}{2} L^2 - PL^2 \right]$$

$$(20) \quad \frac{dy}{dx} \bigg|_{x=L} = - \frac{PL^2}{2EI}$$

negative means the beam is sloping downhill at B

The maximum deflection occurs at point B and is found by substituting $x = L$ into equation (18) as follows

$$(21) \quad y \bigg|_{x=L} = \frac{1}{EI} \left[\frac{P}{6} L^3 - \frac{PL}{2} L^2 \right]$$

$$(22) \quad y \bigg|_{x=L} = - \frac{PL^3}{3EI}$$

negative means the beam deflection is downward at B

