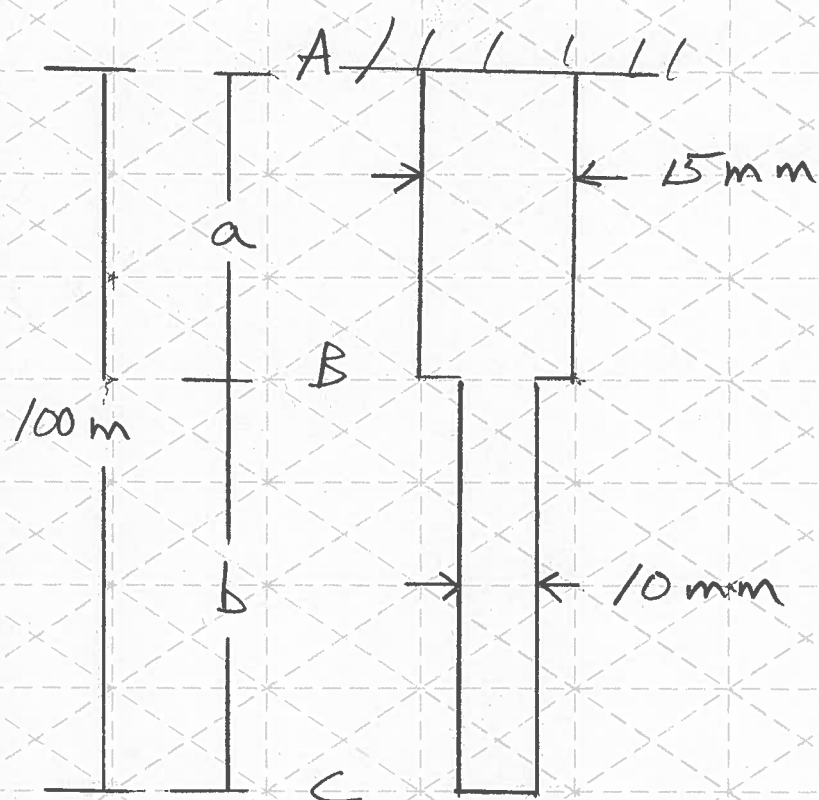


Two brass rods, AB and BC, each of uniform diameter are connected together to form a non uniform rod of total length of 100 m. The rod is supported as shown. Assume that the density of brass is 8470 kg/m^3 . Determine the length of rod AB for which the maximum normal stress in ABC is minimized and what is this value.



$$A_{AB} = \pi \left(\frac{0.015}{2} \right)^2 \quad (1)$$

$$A_{AB} = 1.767 \times 10^{-4} \text{ m}^2 \quad (2)$$

$$A_{BC} = \pi \left(\frac{0.010}{2} \right)^2 \quad (3)$$

$$A_{BC} = 7.854 \times 10^{-5} \text{ m}^2 \quad (4)$$

$$\text{Vol}_{AB} = (1.767 \times 10^{-4} \text{ m}^2)(a) = 1.767 \times 10^{-4} a \text{ m}^3 \quad (5)$$

$$\text{Vol}_{BC} = (7.854 \times 10^{-5} \text{ m}^2)(b) = 7.854 \times 10^{-5} b \text{ m}^3 \quad (6)$$

$$\text{but } b = 100 - a \quad (7)$$

So

$$\text{Vol}_{BC} = 7.854 \times 10^{-5} (100 - a) \quad (8)$$

$$\text{or}$$

$$\underline{Vol_{BC} = 7.854 \times 10^{-3} - 7.854 \times 10^{-5} \text{ a} \text{ m}^3}$$

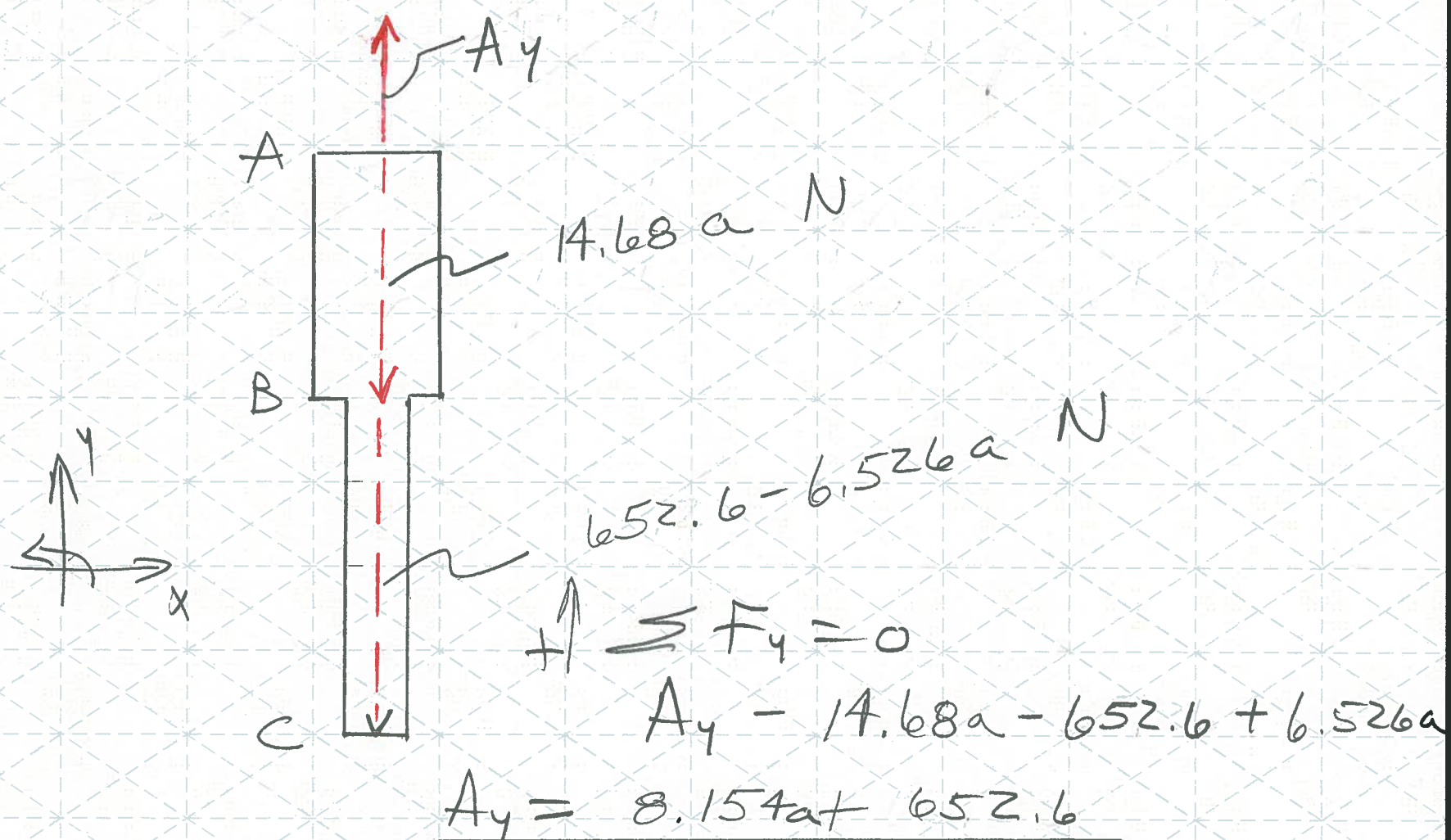
$$W_{AB} = (1.767 \times 10^{-4} \text{ a m}^3) \left(8470 \frac{\text{kg}}{\text{m}^3} \right) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)$$

$$\underline{W_{AB} = 14.68 \text{ a} \frac{\text{kg m}}{\text{s}^2} \text{ or N}}$$

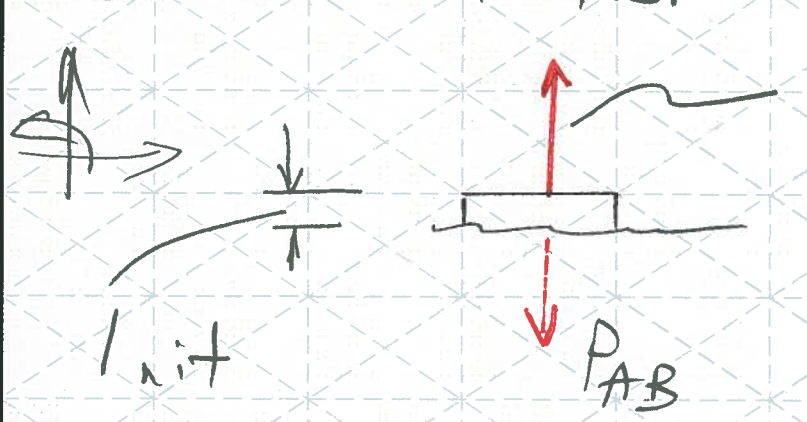
$$W_{BC} = (7.854 \times 10^{-3} - 7.854 \times 10^{-5} \text{ a m}^3) \left(8470 \frac{\text{kg}}{\text{m}^3} \right) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)$$

$$\underline{W_{BC} = 652.6 - 6.526 \text{ a} \frac{\text{kg m}}{\text{s}^2} \text{ or N}}$$

Next Draw a F.B.D. of ABC



The maximum stress in AB occurs at 1_{nit} below A. Make cut and draw F.B.D.



A free body diagram of a section of the bar. A horizontal line represents the bar. A vertical dashed line indicates a cut at a distance 1_{nit} from point A. To the left of the cut, a curved arrow labeled 1_{nit} points downwards. To the right of the cut, a red arrow labeled P_{AB} points downwards. Above the cut, a red arrow points upwards, labeled $8.154a + 652.6$. To the right of the diagram, the following equations are written:

$$\uparrow \sum F_y = 0$$

$$-P_{AB} + 8.154a + 652.6 = 0 \quad (16)$$

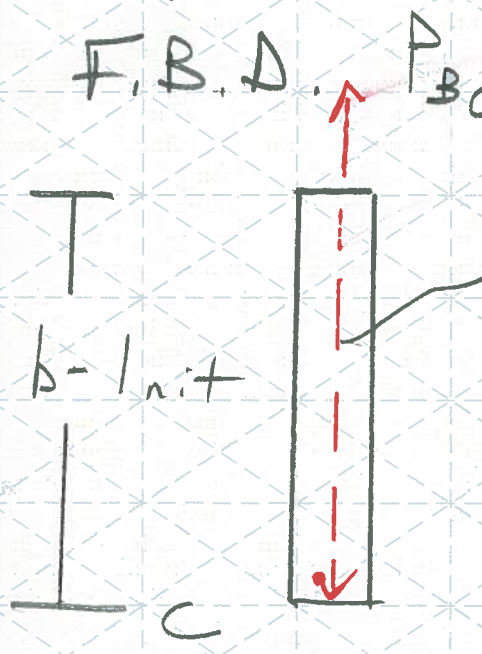
$$P_{AB} = 8.154a + 652.6 \quad (17)$$

$$0 \leq a \leq 100$$

$$\sigma_{AB}^{Max} = \frac{8.154a + 652.6}{1.767 \times 10^{-4} \text{ m}^2} \text{ N} \quad (18)$$

$$\sigma_{AB} = 4.615 \times 10^4 a + 3.693 \times 10^6 \frac{\text{N}}{\text{m}^2 \text{ or Pa}} \quad (19)$$

The maximum stress in BC occurs at 1_{nit} below B. Make cut and draw F.B.D.



A free body diagram of a section of the bar. A vertical line represents the bar. A horizontal dashed line indicates a cut at a distance 1_{nit} from point B. To the left of the cut, a vertical line segment is labeled 1_{nit} . To the right of the cut, a red arrow labeled P_{BC} points downwards. Above the cut, a red arrow points upwards, labeled $652.6 - 6.526a \text{ N}$. To the right of the diagram, the following equations are written:

$$\uparrow \sum F_y = 0$$

$$P_{BC} - 652.6 + 6.526a = 0 \quad (20)$$

$$P_{BC} = 652.6 - 6.526a \quad (21)$$

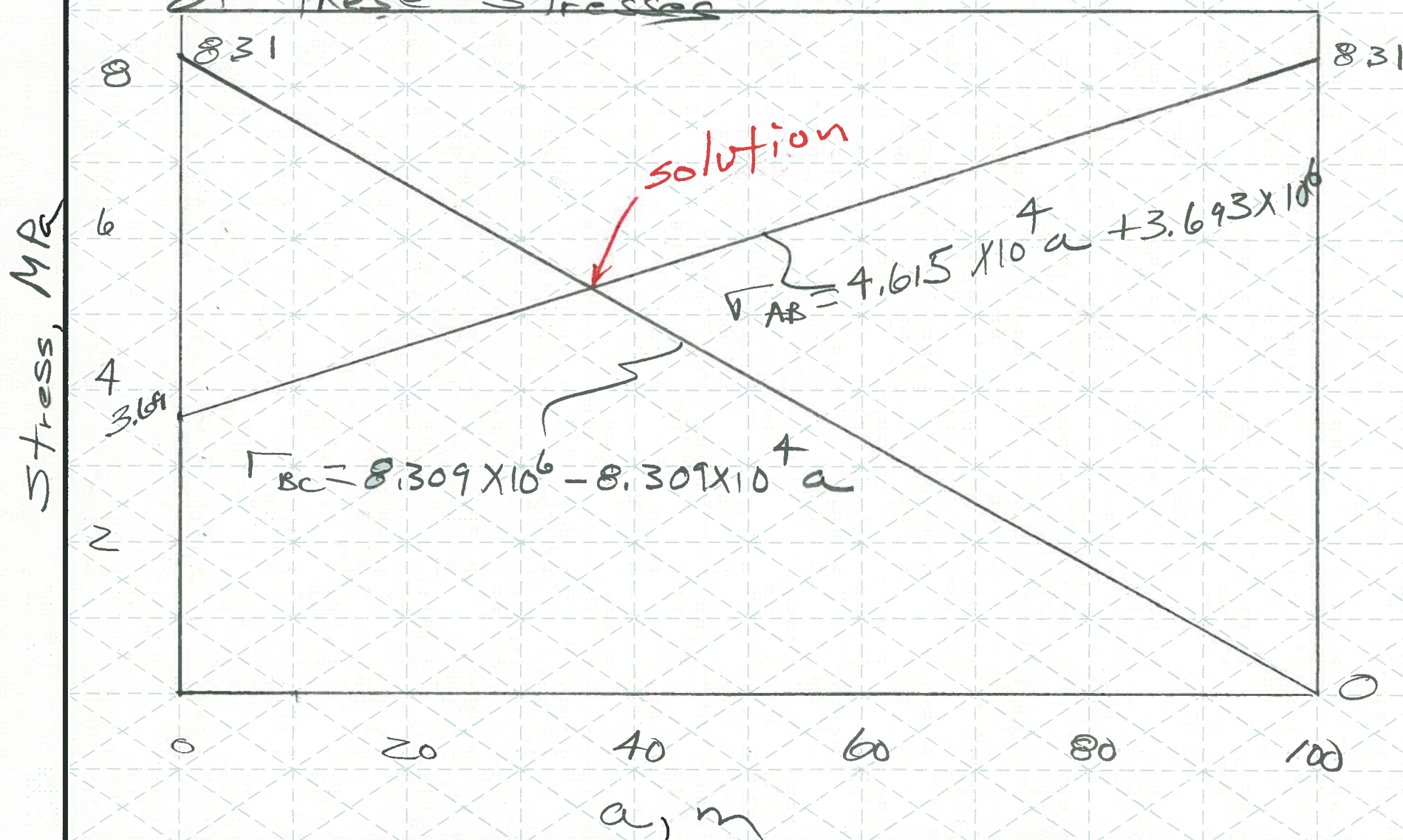
$$\sigma_{BC} = \frac{652.6 - 6.526a}{7.854 \times 10^{-5} \text{ m}^2} \text{ N} \quad (22)$$

$$\sigma_{BC} = 8.309 \times 10^6 - 8.309 \times 10^4 a \frac{\text{N}}{\text{m}^2} \quad (23)$$

The maximum stress in AB ranges linearly from 3.69 MPa to 8.31 MPa as a ranges from 0 to 100.

The maximum stress in BC ranges linearly from 8.31 MPa to 0 MPa as a ranges from 0 to 100.

The following graph shows the variations of these stresses



Evaluation of the above graph quickly leads to the conclusion that the maximum stress in bar ABC is minimized when $\tau_{AB} = \tau_{BC}$ (24)

$$4.615 \times 10^4 a + 3.693 \times 10^6 = 8.309 \times 10^6 - 8.309 \times 10^4 a \quad (25)$$

Solving equation (25) for a yields

$$1.2924 \times 10^5 a = 4.616 \times 10^6 \quad (26)$$

$$\underline{a = 35.7 \text{ m}} \quad (27)$$

Plugging $a = 35.7 \text{ m}$ into either equation (19) or (23) yields

$$\underline{\tau_{AB} = \tau_{BC} = 5.34 \times 10^6 \text{ Pa}} \quad (28)$$