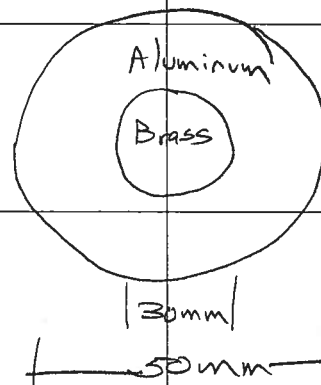
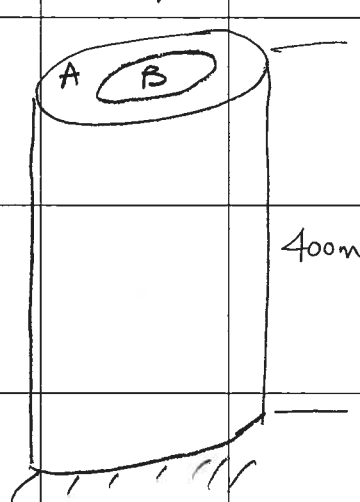


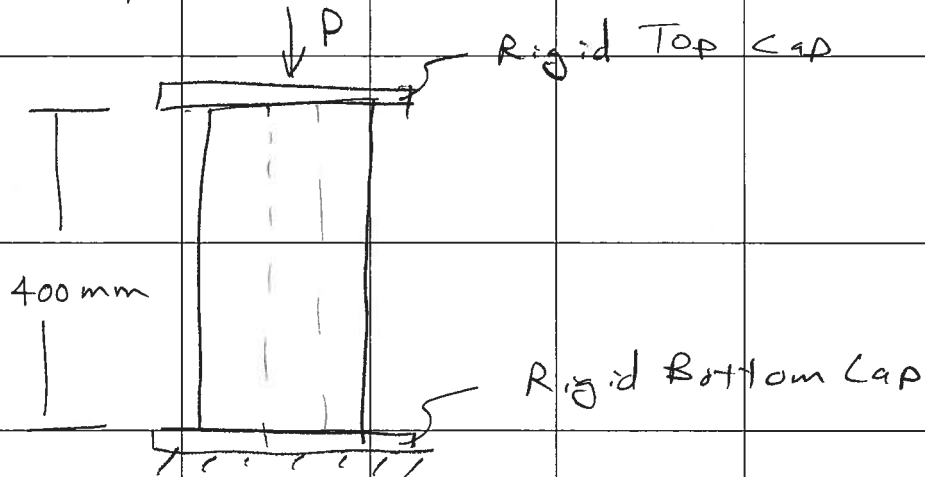
A cylindrical member consists of a brass core inside a hollow aluminum pipe. The brass and aluminum are in intimate contact. The length of the member is 400 mm. The diameter of the core is 30 mm and the outer diameter of the aluminum pipe is 50 mm. The allowable stress for the brass is 120 MPa and the allowable stress for the aluminum is 80 MPa. What is the maximum allowable compressive load, P , that can be applied to the member and what is the corresponding axial deformation.



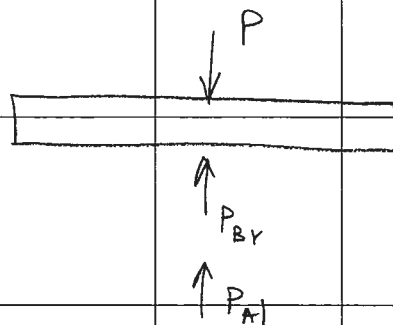
$$E_{Br} = 100 \times 10^9 \frac{N}{m^2}$$

$$E_{Al} = 70 \times 10^9 \frac{N}{m^2}$$

In problems such as this, it is almost always assumed that the assembly has a rigid cap on the top and bottom as shown



Equilibrium Relationship

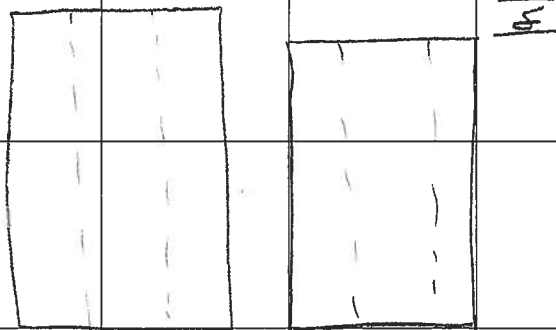


$$+\uparrow \sum F_y = 0$$

$$-P + P_{Br} + P_{A1} = 0 \quad (1)$$

$$P = P_{Br} + P_{A1} \quad (2)$$

Displacement Relationships



Undeformed Deformed

Because the cylindrical member is symmetric and it is loaded in a symmetric manner it can be concluded that the deflection of the brass is equal to the deflection of the aluminum is equal to the overall deflection S .

$$S = S_{Br} = S_{Al} \quad (3)$$

Now we examine S_{Br}

$$S_{Br} = \frac{P_{Br} L_{Br}}{A_{Br} E_{Br}} \quad (4)$$

The area of the brass is given as follows:

$$A_{Br} = \frac{\pi}{4} (.030)^2 = 7.069 \times 10^{-4} \text{ m}^2 \quad (5)$$

So

$$\delta_{Br} = \frac{P_{Br} (.1400 \text{ m})}{(7.069 \times 10^{-4} \text{ m}^2) (100 \times 10^9 \frac{\text{N}}{\text{m}^2})} \quad (6)$$

$$\delta_{Br} = 5.659 \times 10^{-9} P_{Br} \quad (7)$$

$$\text{or } P_{Br} = 1.767 \times 10^8 \delta_{Br}$$

Examine δ_{Al}

$$\delta_{Al} = \frac{P_{Al} L_{Al}}{A_{Al} E_{Al}} \quad (8)$$

The area of the aluminum is given as follows

$$A_{Al} = \frac{\pi}{4} (.050^2 - .030^2) \quad (9)$$

$$A_{Al} = 1.257 \times 10^{-3} \text{ m}^2 \quad (10)$$

So

$$\delta_{Al} = \frac{P_{Al} (.1400 \text{ m})}{(1.257 \times 10^{-3} \text{ m}^2) (70 \times 10^9 \frac{\text{N}}{\text{m}^2})} \quad (11)$$

$$\delta_{Al} = 4.546 \times 10^{-9} P_{Al} \quad (12)$$

$$\text{or } P_{Al} = 2.200 \times 10^8 \delta_{Al}$$

Next we plug the displacement information into the equilibrium information yielding.

$$P = 1.767 \times 10^8 \delta + 2.200 \times 10^8 \delta \quad (13)$$

$$P = 3.967 \times 10^8 \delta \quad (14)$$

$$\text{or} \\ \delta = 2.521 \times 10^{-9} P \quad (15)$$

Thus

$$P_{Br} = 1.767 \times 10^8 (2.521 \times 10^{-9} P) \quad (16)$$

$$P_{Br} = .445 P \quad (17)$$

and

$$P_{Al} = (2.200 \times 10^8) (2.521 \times 10^{-9} P) \quad (18)$$

$$P_A = .555 P \quad (19)$$

There are two independent conditions that cannot be violated:

$$\tau_{A1} \leq 80 \times 10^6 \frac{\text{N}}{\text{m}^2} \text{ and } \tau_{B1} \leq 120 \times 10^6 \frac{\text{N}}{\text{m}^2}$$

Examine τ_{A1} first

$$P_{A1_{\max}} = A_{A1} \tau_{A1_{\text{allowable}}} \quad (20)$$

$$P_{A1_{\max}} = (1.257 \times 10^{-3} \text{ m}^2) (80 \times 10^6 \frac{\text{N}}{\text{m}^2}) \quad (21)$$

$$P_{A1_{\max}} = 100,560 \text{ lb} \quad (22)$$

So from equation (19)

$$P_{\text{allow}} = \frac{P_{A1_{\max}}}{.555} \quad (23)$$

$$P_{\text{allow}} = \frac{100560}{.555} \quad (24)$$

$$P_{\text{allow}} = 181,189 \text{ lbs} \quad (25)$$

Maybe

Examine $\sigma_{Br \text{ next}}$

$$P_{Br \max} = A_{Br} \sigma_{\text{allowable}} \quad (26)$$

$$P_{Br \max} = (7.069 \times 10^{-4} \text{ m}^2) (120 \times 10^6 \frac{\text{N}}{\text{m}^2}) \quad (27)$$

$$P_{Br \max} = 84,828 \text{ lb} \quad (28)$$

So from equation

$$P_{\text{allow}} = \frac{P_{Br \text{ allow}}}{1.445} \quad (29)$$

$$P_{\text{allow}} = \frac{84828}{1.445} \quad (30)$$

$$P_{\text{allow}} = 190,625 \text{ lbs} \quad (31)$$

Maybe

Finally

$$181,189 < 190,625 \quad (32)$$

So

The maximum allowable load is
181,189 lbs

The corresponding deflection
from equation (15) is

$$\delta = (2.521 \times 10^{-9})(181, 189) \quad (33)$$

$$\delta = 4.57 \times 10^{-4} \text{ in} \quad (34)$$