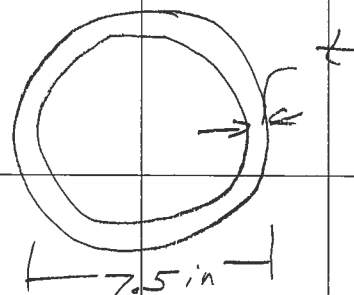


A hollow, circular, steel column is subjected to a compressive load as shown. The column has a length $L = 8.0 \text{ ft}$ and outside diameter $d = 7.5 \text{ in.}$ The load $P = 85 \text{ k.}$ The allowable stress is 7000 psi and the allowable shortening of the column is 0.02 in. What is the minimum required wall thickness $t_{\min}$?

Rigid End Cap P

Assume $E_{\text{steel}} = 30 \times 10^6 \text{ psi}$



There are two design conditions that must be met: the allowable stress condition and the maximum axial deflection condition.

First Check Allowable stress

calculate cross-sectional area, A_0

$$A_0 = \pi \left(\left(\frac{7.5}{2} \right)^2 - \left(\frac{7.5}{2} - t \right)^2 \right) \text{ in}^2$$

$$A_0 = \pi (14.06 - (14.06 - 7.5t + t^2)) \text{ in}^2$$

$$A_0 = \pi (7.5t - t^2) \text{ in}^2$$

$$\tau_{\text{Allow}} = \frac{P}{A_0}$$

$$7000 \frac{\text{lb}}{\text{in}^2} = \frac{85000 \text{ lb}}{\pi (7.5t - t^2) \text{ in}^2}$$

From which we find
that $t = \underline{.556 \text{ in}}$

Next Check Allowable Deflection

$$\delta_{\text{Allow}} = \frac{PL}{AE}$$

$$0.020 \text{ in} = \frac{(85000 \text{ lb})(96 \text{ in})}{\pi (7.5t - t^2) \text{ in}^2 (30 \times 10^6 \frac{\text{lb}}{\text{in}^2})}$$

$$0.020 = \frac{.272}{\pi (7.5t - t^2)}$$

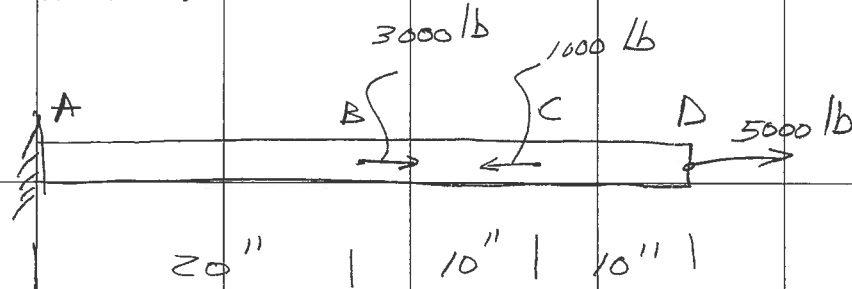
from which we find
that $t = \underline{.63 \text{ in}}$

Because $.63 > .556$

The deflection criterion controls

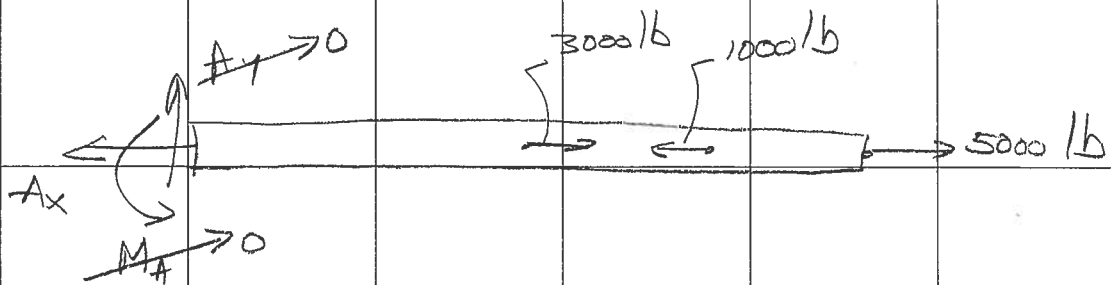
Therefore, the minimum
required wall thickness is
 0.63 in.

A steel bar ABCD has a constant cross sectional area of $.5 \text{ in}^2$ and is loaded as shown.



Determine the deflection of point D assuming that the modulus of elasticity of steel is $30 \times 10^6 \text{ psi}$.

First it is necessary to calculate the reactions at A



$$\Rightarrow \sum F_x = 0$$

$$-A_x + 3000 - 1000 + 5000 = 0$$

$$A_x = 7000 \text{ lbs} \leftarrow$$

Next it is observed that the bar must be divided into three different sections where the material properties, cross-sectional area, and modulus of elasticity remain constant. Then the total deflection at D is given as follows:

$$\delta_D = \delta_{AB} + \delta_{BC} + \delta_{CD}$$

Then each of the individual deflection components must be independently evaluated as follows:

δ_{AB}



$P_{AB} \quad \rightarrow \quad \sum F_x = 0$

$-7000 + P_{AB} = 0$

$P_{AB} = 7000 \text{ lbs}$

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{A_{AB} E_{AB}}$$

$$\delta_{AB} = \frac{(7000 \text{ lbs})(20 \text{ in})}{(0.5 \text{ in}^2)(30 \times 10^6 \frac{\text{lb}}{\text{in}^2})}$$

$$\delta_{AB} = .00933 \text{ in.}$$

σ_{BC}		Note that all internal forces are assumed to be the same. In this case tension
	$\rightarrow \sum F_x = 0$	
	$-7000 + 3000 + P_{BC} = 0$	
	$P_{BC} = 4000 \text{ lbs}$	
	$\sigma_{BC} = \frac{P_{BC} L_{BC}}{A_{BC} F_{BC}}$	
	$\sigma_{BC} = \frac{(4000 \text{ lbs})(10 \text{ in})}{(0.5 \text{ in}^2)(30 \times 10^6 \frac{\text{lb}}{\text{in}^2})}$	
	$\sigma_{BC} = .00267 \text{ in}$	
σ_{CD}		
	$\rightarrow \sum F_x = 0$	
	$-7000 + 3000 - 1000 + P_{CD} = 0$	
	$P_{CD} = 5000 \text{ lbs}$	

$$\delta_{CD} = \frac{P_{CD} L_{CD}}{A_{CD} E_{CD}}$$

$$\delta_{CD} = \frac{(5000 \text{ lbs})(10 \text{ in})}{(0.50 \text{ in}^2)(30 \times 10^6 \frac{\text{lb}}{\text{in}^2})}$$

$$\delta_{CD} = .00333 \text{ in.}$$

Finally

$$\delta_D = \delta_{AB} + \delta_{BC} + \delta_{CD}$$

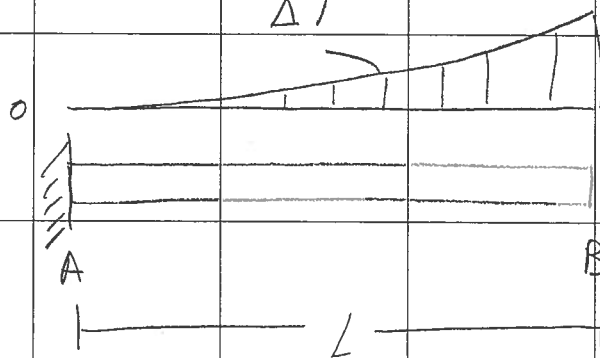
$$\delta_D = .00933 + .00267 + .00333$$

$$\delta_D = .01533 \text{ in}$$

A bar AB of length L is heated non uniformly so that the temperature increase ΔT at a distance x from end A is given by the expression

$$\Delta T = \Delta T_B \frac{x^2}{L^2}$$

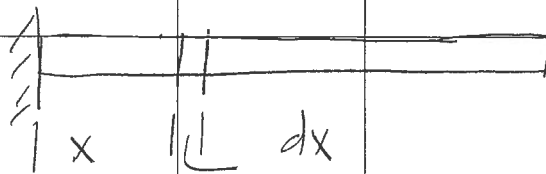
where ΔT_B is the increase in temperature at end B of the bar as shown



Determine a formula for the increase in length of the bar AB assuming that the thermal coefficient of expansion is α .

Because the temperature increase ΔT varies as a function of position on the bar it is necessary to solve the problem by integration.

To do this a differential element located a distance x from A is identified as follows:



dx

The expansion of this differential element, $d\delta$, is given as follows:

$$d\delta = \alpha \Delta T dx$$

The total expansion or change in length of the bar, δ_{AB} , is given as follows:

$$\delta_{AB} = \int_0^L \alpha \Delta T dx$$

Then substituting in the given expression for the temperature increase we get

$$\sigma_{AB} = \int_0^L \alpha \Delta T_B \frac{x^2}{L^2} dx$$

Removing the constants from the integral we get

$$\sigma_{AB} = \frac{\alpha \Delta T_B}{L^2} \int_0^L x^2 dx$$

which becomes

$$\sigma_{AB} = \frac{\alpha \Delta T_B L}{3}$$

Thus

$$\boxed{\sigma_{AB} = \frac{\alpha \Delta T_B L}{3}}$$