

Q - "the 1st moment with respect to the neutral axis of a portion of the cross-section located either above or below the point" of interest on the cross-section.

Beer and Johnston
2002

$$Q = \int_{\text{Area Above Centroid}} y \, dA \quad \left\{ \begin{array}{l} \text{differential} \\ \text{Area} \end{array} \right.$$

Q is maximum when evaluated at the centroid.

distance from the neutral axis of the total cross-section to the centroid of the differential area. This distance is measured positive upward.

Or

$$Q = \sum_{\text{Area Above Centroid}} A_i y_i \quad \left\{ \begin{array}{l} \text{Discrete areas above} \\ \text{the point of interest} \end{array} \right.$$

distance from the neutral axis of the total cross-section to the centroid of the ⁱth discrete area

If you choose to use the area below the point everything is the same except that a minus sign must be placed before the integral or summation as follows.

$$Q = - \int_{\substack{\text{Area} \\ \text{Below Centroid}}} y \, dA$$

or

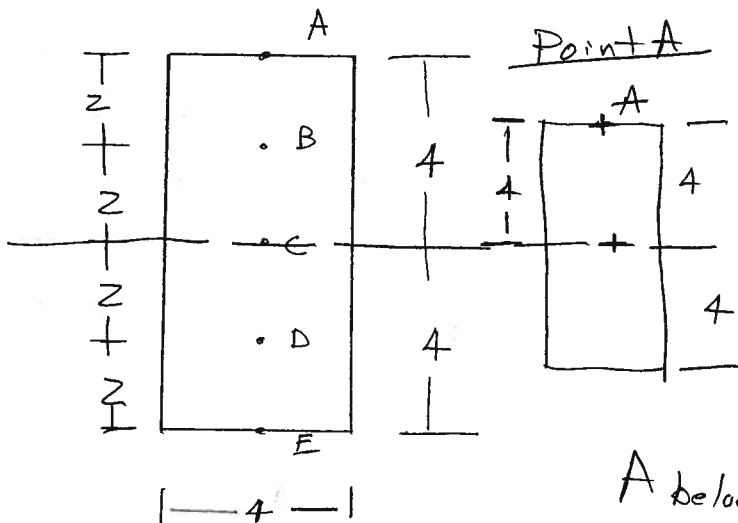
$$Q = - \sum_{\substack{\text{Area} \\ \text{Below Centroid}}} A_i y_i$$

Where everything is defined as above.

When a cross-section is cut into two pieces — one will have a centroid that is located above the cross-section that is called the area above the centroid. The other piece will have a centroid that is located below the cross-section centroid that is called the area below the centroid.

Rectangle Example

Centroid is obviously in the center as shown



$$A_{\text{above}} = 0$$

$$Q_A = 0(4)$$

$$\underline{Q_A = 0}$$

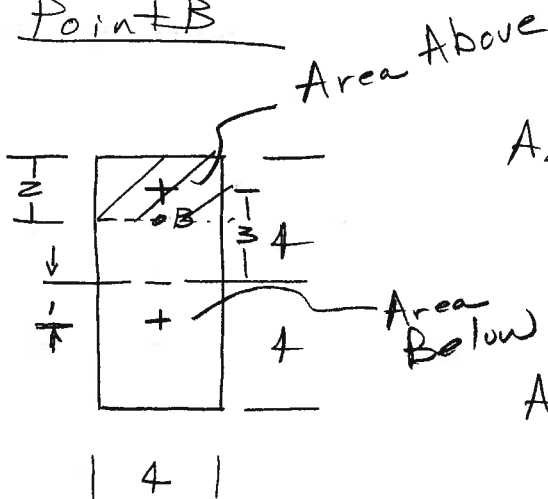
or

$$A_{\text{below}} = (4)(8) = 32$$

$$Q_A = (32)(0)$$

$$\underline{Q_A = 0}$$

Point B



$$A_{\text{above}} = (2)(4) = 8$$

$$Q_B = 8(3)$$

$$\underline{Q_B = 24}$$

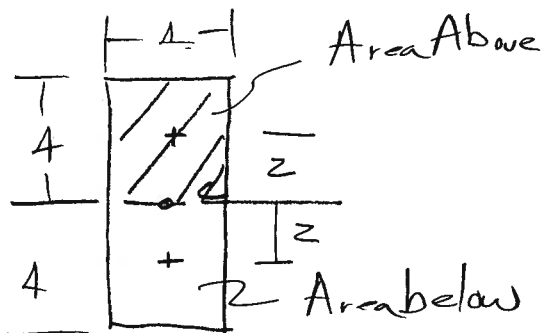
or

$$A_{\text{below}} = (6)(4) = 24$$

$$Q_B = -(24)(-1)$$

$$\underline{Q_B = 24}$$

Point C



$$A_{\text{above}} = (4)(4) = 16$$

$$Q_C = 16(z)$$

$$Q_C = 32$$

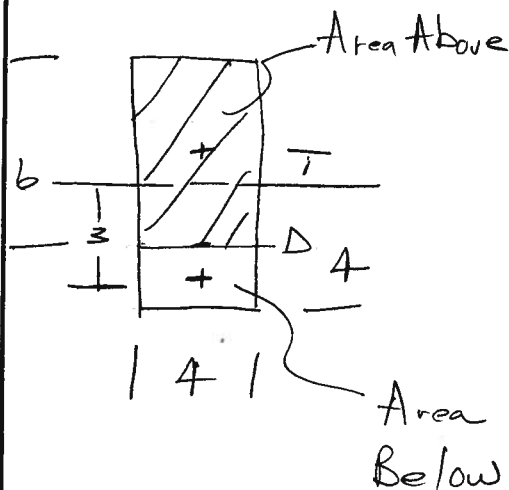
or

$$A_{\text{below}} = (4)(4) = 16$$

$$Q_C = -16(-z)$$

$$Q_C = 32$$

Point D



$$A_{\text{above}} = (6)(4) = 24$$

$$Q_D = (24)(1)$$

$$Q_D = 24$$

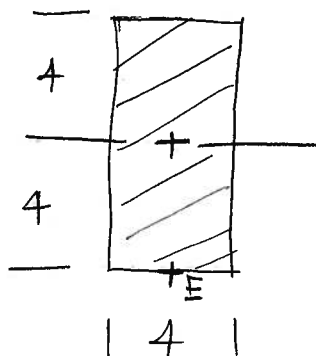
or

$$A_{\text{below}} = (3)(4) = 12$$

$$Q_D = -(12)(-2)$$

$$Q_D = 24$$

Point E



$$A_{\text{Above}} = (8)(4) = 32$$

$$Q_E = (32)(0)$$

$$Q_E = 0$$

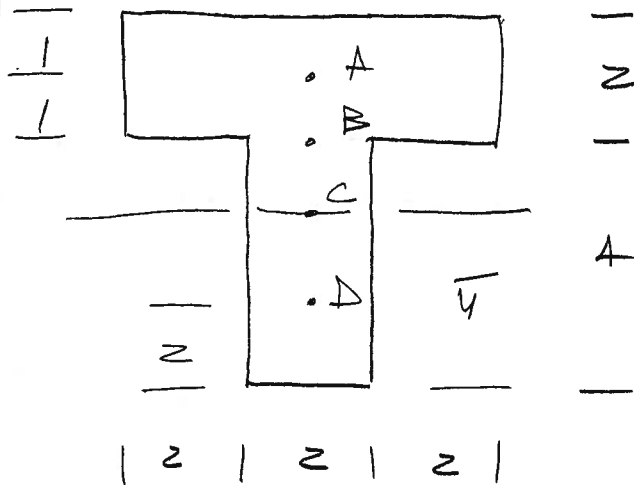
or

$$A_{\text{Below}} = 0$$

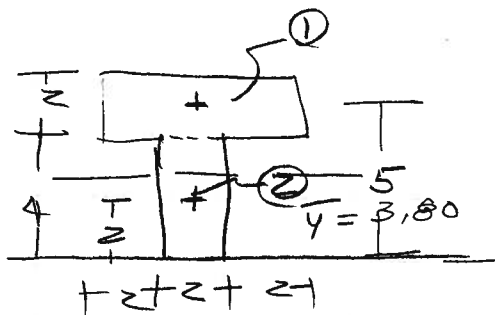
$$Q_E = (0)(4)$$

$$Q_E = 0$$

T Example



First calculate location of centroid.



$$\bar{y} = \frac{(6)(2)(5) + (2)(4)(2)}{(6)(2) + (2)(4)}$$

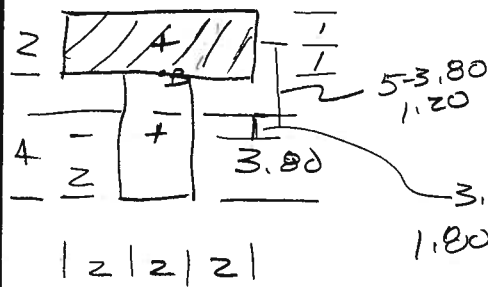
$$\bar{y} = 3.80$$

Point A

$5.5 - 3.80 = 1.70$
 $A_{\text{Above}} = (6)(1) = 6$
 $4.5 - 3.80 = .7$
 $Q_A = (6)(1)(1.7)$
 $Q_A = 10.20$
 or
 $A_{\text{Below}} = (6)(1) + (2)(4) = 14$
 $Q_A = -[(2)(4)(-1.80)] + (6)(1)(1.7)$
 $Q_A = 10.20$

These two values must have opposite signs because one is above Centroid and one is below.

Point B



$$A_{\text{Above}} = 6(2) = 12$$

$$Q_B = (6)(2)(1.2)$$

$$Q_B = 14.40$$

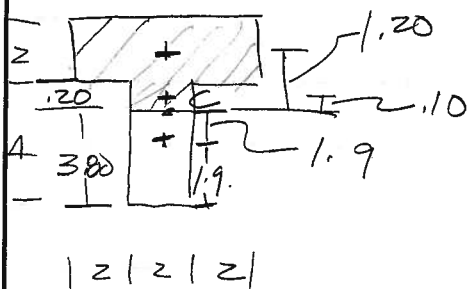
or

$$A_{\text{Below}} = (2)(4) = 8$$

$$Q_B = -(2)(4)(-1.80)$$

$$Q_B = 14.40$$

Point C



$$A_{\text{Above}} = (6)(2) + (2)(2) = 12.40$$

$$Q_C = (6)(2)(1.20) + (2)(2)(1.10)$$

$$Q_C = 14.44$$

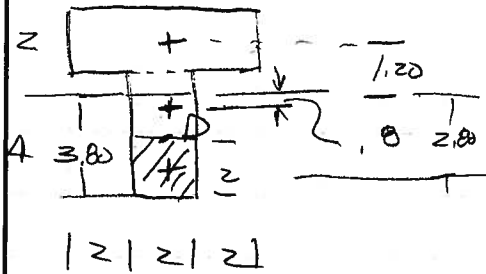
or

$$A_{\text{Below}} = (2)(3.80) = 7.60$$

$$Q_C = -(2)(3.80)(-1.9)$$

$$Q_C = 14.44$$

Point D



$$A_{\text{above}} = (6)(2) + (2)(2) = 16$$

$$Q_D = (6)(2)(1.20) + (2)(2)(-2.80)$$

$$Q_D = 11.20$$

or

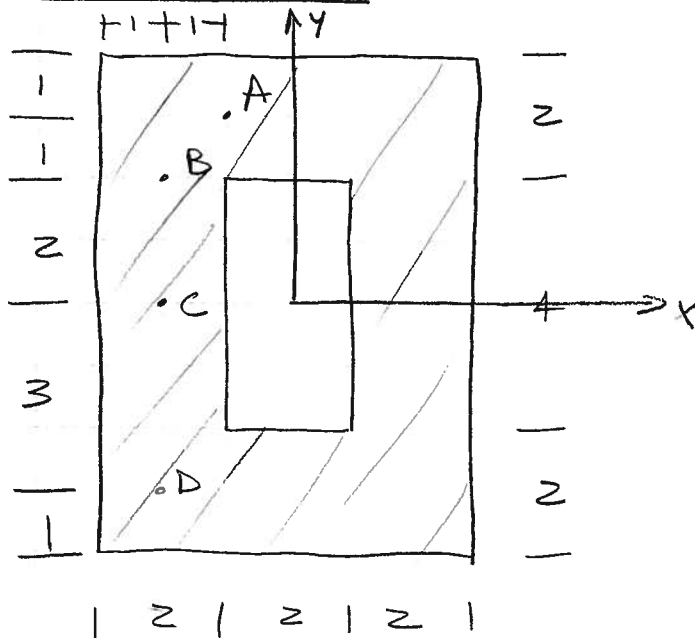
$$A_{\text{below}} = (2)(2) = 4$$

$$Q_D = -(2)(2)(2.80)$$

$$Q_D = 11.20$$

Opposite
signs

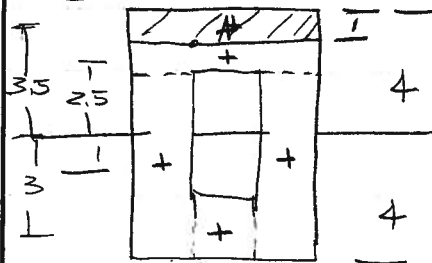
Box Example



By symmetry it is clear that the centroid is in the middle as shown.

Point A

Point A can be interpreted in two ways. First we will assume a horizontal cut as shown



Area above Centroid

$$Q_{A_{\text{Above}}} = + (6)(1)(3.5)$$

$$Q_{A_{\text{Above}}} = \underline{21}$$

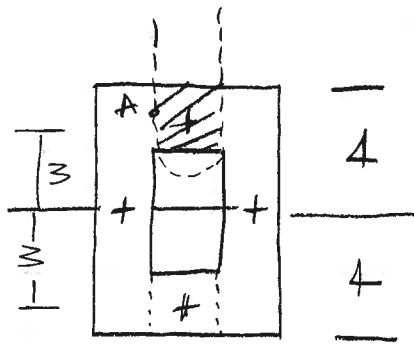
or

Area below Centroid

$$Q_{C_{\text{Below}}} = - [(6)(1)(2.5) + (2)(6)(-1) + (2)(6)(-1) + (2)(2)(-3)]$$

$$Q_{C_{\text{Below}}} = \underline{21}$$

Next we will assume a vertical cut at point A. It should be noted that a simple vertical cut through A does not isolate two distinct areas. Therefore, a second cut is required to isolate two distinct areas. In most problems where two cuts are required, it is desirable to use a second cut based on symmetry as shown.



$$|2|2|2|$$

Area above Centroid

$$Q_{A \text{ Above}} = + (2)(2)(3)$$

$$Q_{A \text{ Above}} = 12$$

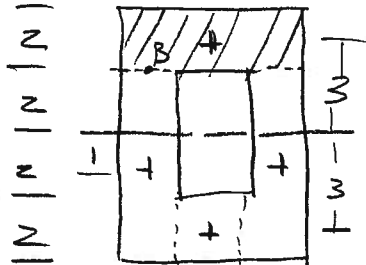
or

Area below Centroid

$$Q_{A \text{ Below}} = - \left[(2)(2)(0) + (2)(2)(0) - (2)(2)(-3) \right]$$

$$Q_{A \text{ Below}} = 12$$

Point B



$$|z|z|z|$$

As with the vertical cut at A, a single horizontal cut at B does not isolate two distinct areas. Therefore, a second is required to isolate two distinct areas. Therefore a similar cut on the right side of the element is made to develop a symmetric area as shown

Area above Centroid

$$Q_{A \text{ Above}} = (6)(2)(3)$$

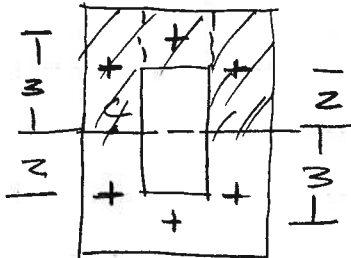
$$Q_{A \text{ Above}} = \underline{36}$$

Area below Centroid

$$Q_{A \text{ Below}} = -[(2)(6)(-1) + (2)(6)(-1) + (2)(2)(-3)]$$

$$Q_{A \text{ Below}} = \underline{36}$$

Point C



Once again it is necessary to cut completely across the cross-section to isolate two distinct areas.

Area above Centroid

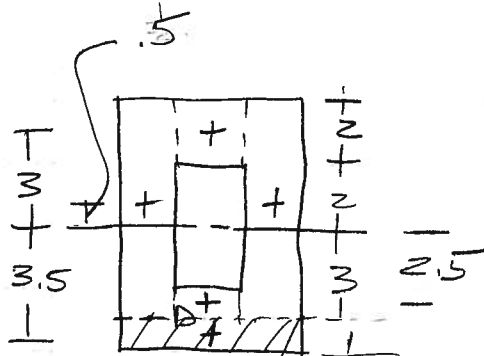
$$Q_{C \text{ Above}} = [(2)(4)(2) + (2)(4)(2) + (2)(2)(3)]$$

$$Q_{C \text{ Above}} = \underline{44}$$

Area below Centroid

$$Q_{C \text{ Below}} = - [(2)(4)(2) + (2)(4)(-2) + (2)(2)(-3)]$$

$$Q_{C \text{ Below}} = \underline{44}$$



Area above Centroid

$$Q_{\text{Above}} = [(2)(7)(.5) + (2)(7)(.5) + (2)(2)(3) + (2)(1)(-25)]$$

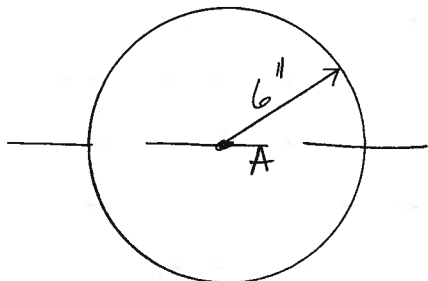
$$Q_D = \underline{21}$$

Area below Centroid

$$\Phi_D|_{\text{below}} = -[(6)(1)(-3,5)]$$

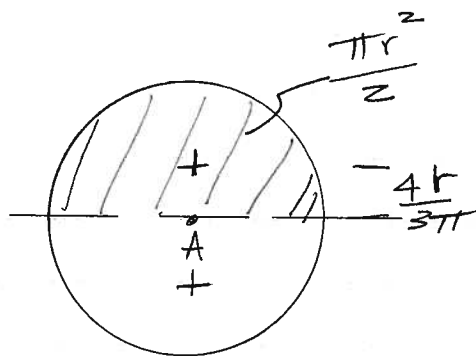
$$Q_{\Delta \text{ Below}} = \underline{21}$$

Circle Example



By symmetry it is obvious that the centroid is located as shown.

Point A



Area above Centroid

$$Q_{A \text{ Above}} = \frac{\pi (6)^2}{2} \frac{4(6)}{3\pi}$$

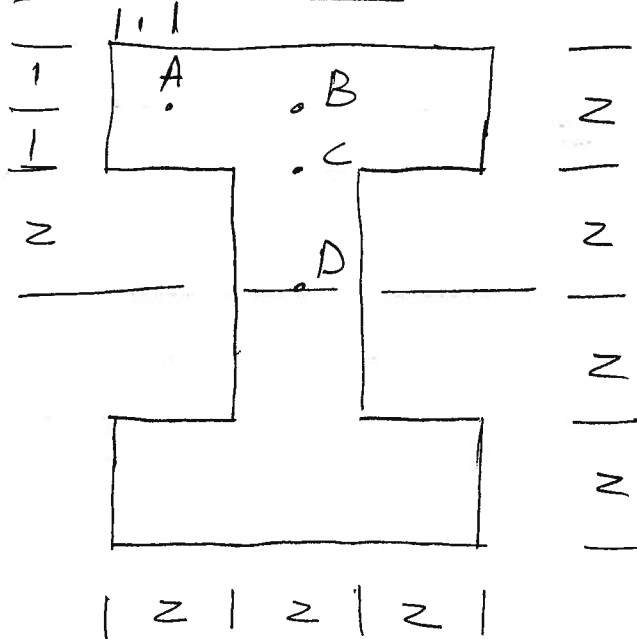
$$Q_{A \text{ Above}} = \underline{144}$$

Area below Centroid

$$Q_{A \text{ Below}} = - \left[\frac{\pi (6)^2}{2} \left(- \frac{4(6)}{3\pi} \right) \right]$$

$$Q_{A \text{ Below}} = \underline{144}$$

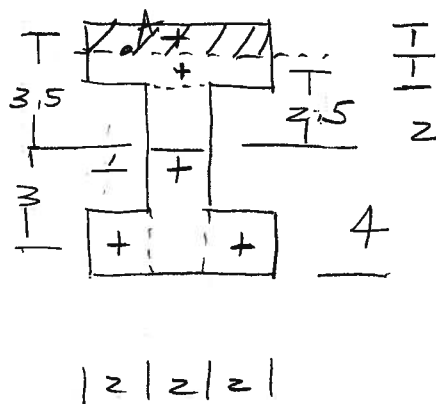
I Example



By symmetry it is clear that the centroid is in the middle as shown

Point A

Point A can be interpreted in two ways. First we will assume a horizontal cut.



Area above centroid

$$Q_{A_{\text{Above}}} = (6)(1)(3.5)$$

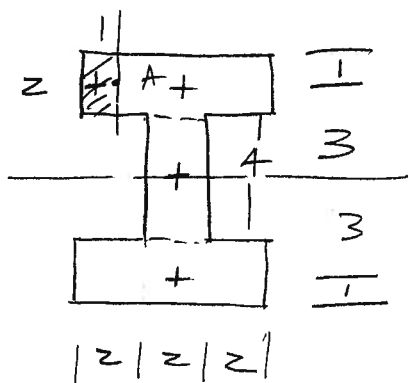
$$Q_{A_{\text{Above}}} = 21$$

Area below Centroid

$$Q_{A_{\text{Below}}} = - \left[(6)(1)(2.5) + (2)(6)(-1) + (2)(2)(-3) + (2)(2)(-3) \right]$$

$$Q_{A_{\text{Below}}} = \underline{21}$$

Next we will assume a vertical cut as shown.



Area above centroid

$$Q_{A_{\text{Above}}} = (1)(2)(3)$$

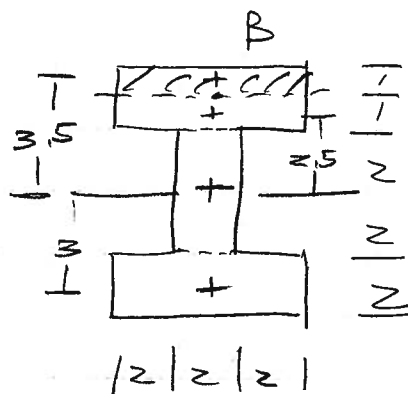
$$Q_{A_{\text{Above}}} = \underline{6}$$

Area below centroid

$$Q_{A_{\text{Below}}} = - \left[(5)(2)(3) + (2)(4)(0) + (6)(2)(-3) \right]$$

$$Q_{A_{\text{Below}}} = \underline{6}$$

Next we will assume a horizontal cut as shown.



Area above Centroid

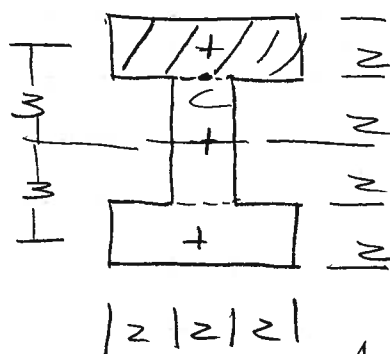
$$Q_{B, \text{Above}} = (6)(1)(3.5)$$

$$Q_{B, \text{Above}} = \underline{21}$$

Area below Centroid

$$Q_{B, \text{Below}} = -[(6)(1)(2.5) + (2)(6)(0) + (6)(2)(-3)]$$

$$Q_{B, \text{Below}} = \underline{21}$$



Area above Centroid

$$Q_{C, \text{Above}} = (6)(2)(3)$$

$$Q_{C, \text{Above}} = \underline{36}$$

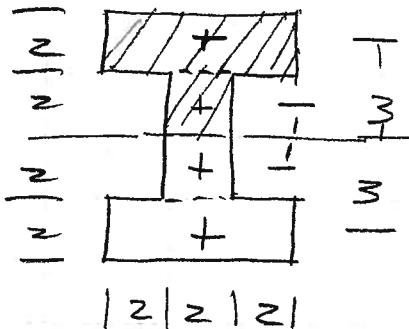
Area below Centroid

$$Q_{C, \text{Below}} = -[2(4)(0) + (6)(2)(-3)]$$

$$Q_{C, \text{Below}} = \underline{36}$$

Point D

Area above Centroid



$$Q_{D \text{ Above}} = (6)(2)(3) + (2)(2)(1)$$

$$Q_{D \text{ Above}} = \underline{40}$$

Area below Centroid

$$Q_{D \text{ Below}} = -[(2)(2)(-1) + (6)(2)(-3)]$$

$$Q_{D \text{ Below}} = \underline{40}$$