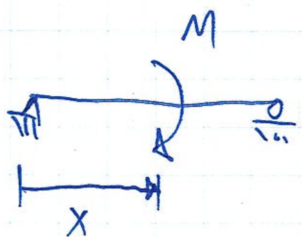
SINGULARITY FUNCTION → Like a light switch that turns the light on (bad at turning the light off)

$\langle X-a \rangle^n$ — if positive → $(X-a)^n$; if negative → 0

- For Integration, treat "$\langle X-a \rangle$" as "X" — NEVER EXPAND
- Always work left-to-right
- Always write moment eqn for the last cut on beam (farthest right eqn)

SPECIAL CASE:

1) External Moment:



"$\langle X-L \rangle$" is effectively a distance (ie ft/in/mm)

$\therefore \langle X-L \rangle^0$ for moment

2) Singularity fcts turn load on, they can't ~~the~~ turn them off

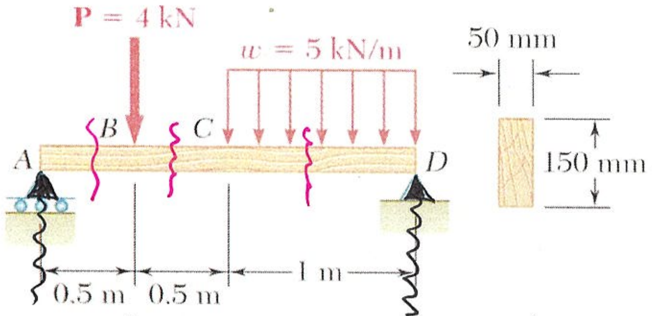
→ we overcome this by superposition

All original material presented was created by Michael S. Brackin, Ph.D., P.E. and is copyrighted 2020. No Part or portion of this material may be copied, transmitted or otherwise used without the express written consent of Michael S. Brackin, Ph.D., P.E.

Use the double integration method with singularity functions as discussed in class to determine the deflection, δ_C, and the slope, θ_A, on the beam loaded as shown. Assume that E= 12 GPa for the timber beam. Enter your results in the blanks provided being sure to include the proper signs.

4-22-2020

3/7

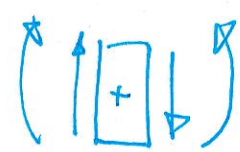


Singularity fctns.

< >

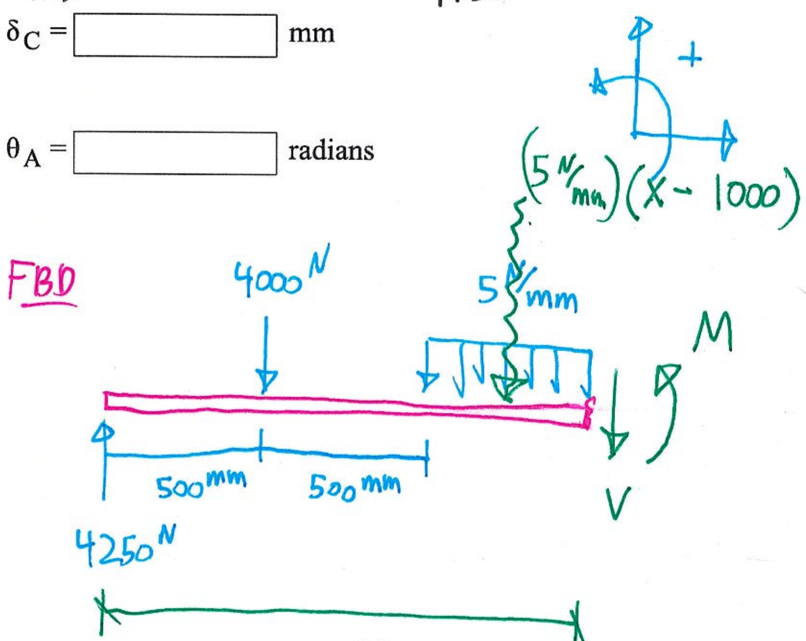
$$I = \frac{1}{12} bh^3 = \frac{1}{12}(50^{mm})(150^{mm})^3 = 14062500 \, mm^4$$

V&M Sign Conv



$\delta_C =$ ______ mm

$\theta_A =$ ______ radians



Resultant

Mom. Arm

$$\circlearrowleft + \Sigma M_{cur} = M - 4250^N(X) + 4000^N \langle X - 500^{mm}\rangle + \underbrace{5\langle X-1000\rangle}_{\text{Resultant}} \, \underbrace{\frac{\langle X-1000\rangle}{2}}_{\text{Mom. Arm}} = 0$$

$$M = -2.5\langle X-1000\rangle^2 - 4000\langle X-500\rangle + 4250X$$

$$EI\frac{d^2y}{dx^2} = M = -2.5\langle X-1000\rangle^2 - 4000\langle X-500\rangle + 4250X$$

$$EI\frac{dy}{dx} = -\frac{2.5}{3}\langle X-1000\rangle^3 - \frac{4000}{2}\langle X-500\rangle^2 + \frac{4250}{2}X^2 + C_1$$

$$EI\,y = \frac{-2.5}{3(4)}\langle X-1000\rangle^4 - \frac{4000}{2(3)}\langle X-500\rangle^3 + \frac{4250}{2(3)}X^3 + C_1X + C_2$$

BNDY COND: $[x=0;\ y=0]$

$$\cancel{EI}(0) = -\frac{2.5}{12}\cancelto{0}{\langle \underbrace{0-1000}_{-1000}\rangle}^4 - \frac{4000}{6}\cancelto{0}{\langle \underbrace{0-500}_{-500}\rangle}^3 + \cancel{\frac{4250}{6}\,0^3} + \cancel{C_1(0)} + C_2$$

$$0 = 0 - 0 + 0 + 0 + C_2 \Rightarrow C_2 = 0$$

$[x = 2000\ \text{mm},\ y=0]$

$$EI(0) = -\frac{2.5}{12}\langle \cancelto{1000}{2000-1000}\rangle^4 - \frac{4000}{6}\langle \cancelto{1500}{2000-500}\rangle^3 + \frac{4250}{6}(2000)^3 + C_1(2000)$$

$$C_1 = -1.604E9$$

$$y = \frac{1}{EI}\left[\frac{-2.5}{12}\langle x-1000\rangle^4 - \frac{4000}{6}\langle x-500\rangle^3 + \frac{4250}{6}(x)^3 - 1.604E9\,x\right]$$

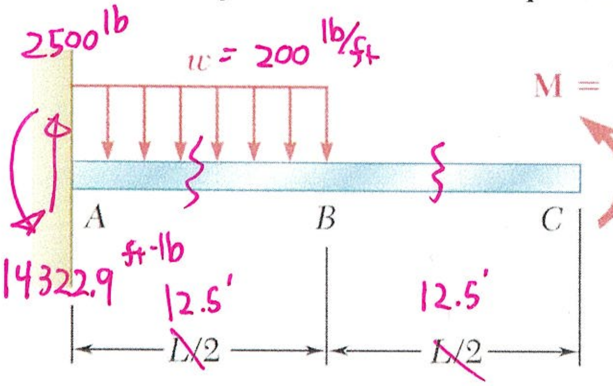
$\delta_B \Rightarrow x = 500$

$$y = \frac{1}{(12E3\ \text{MPa})(14062500)}\left[\frac{-2.5}{12}\cancelto{0}{\langle 500-1000\rangle}^4 - \frac{4000}{6}\cancelto{0}{\langle 500-500\rangle}^3 + \frac{4250}{6}(500)^3 - 1.604E9(500)\right] = -4.23\ \text{mm}$$

All original material presented was created by Michael S. Brackin, Ph.D., P.E. and is copyrighted 2020. No Part or portion of this material may be copied, transmitted or otherwise used without the express written consent of Michael S. Brackin, Ph.D., P.E.

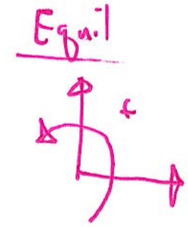
4/7

Use the method of superposition as discussed in class to determine the deflection, δ_C, and the slope, θ_C on the beam loaded as shown. Assume w_o = 200 lb/ft, L = 25 ft, E= 29,000 ksi, and I = 60 in^4. Enter your results in the blanks provided being sure to include the proper signs.

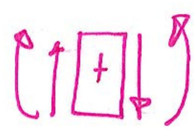


$$M = \frac{wL^2}{24} = \frac{200(12.5)^2}{24} = 1302.1 \text{ ft-lb}$$

Equil



V&M Sign

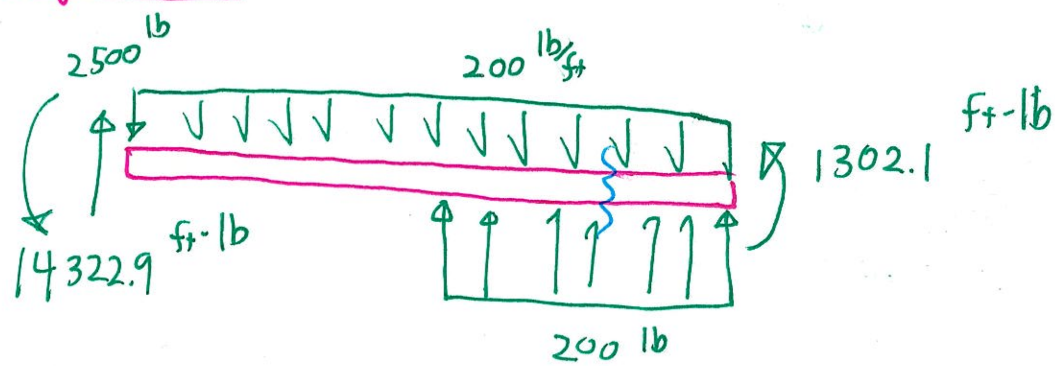


δ_C = ______ in.

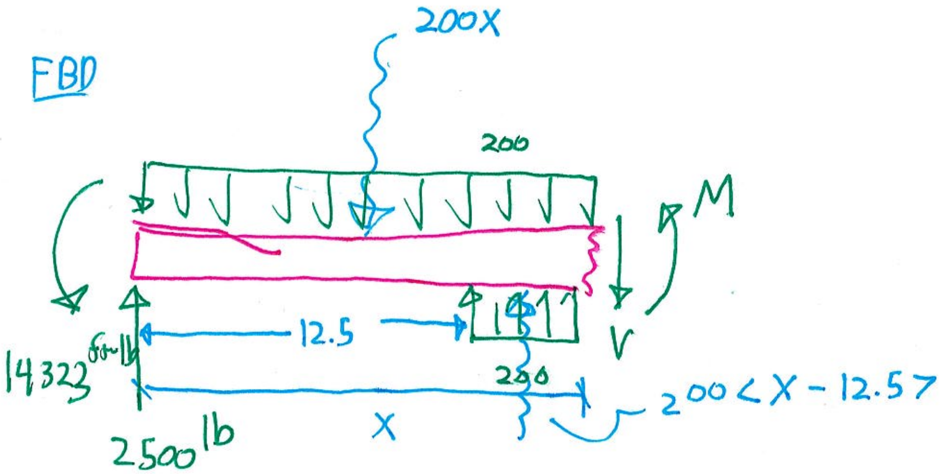
Should be 25 ft throughout

θ_C = ______ radians

Equiv: Beam

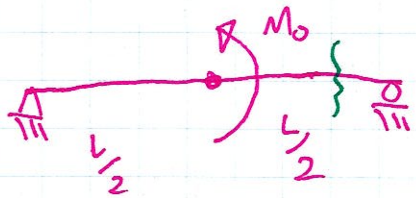


FBD



$$\overset{+}{\curvearrowleft}\sum M_{cut} = M + \underbrace{200x}_{R}\underbrace{\left(\frac{x}{2}\right)}_{d} - 2500(x) + 14323 - \underbrace{200\langle x-12.5\rangle}_{R}\frac{\langle x-12.5\rangle}{2} = 0$$

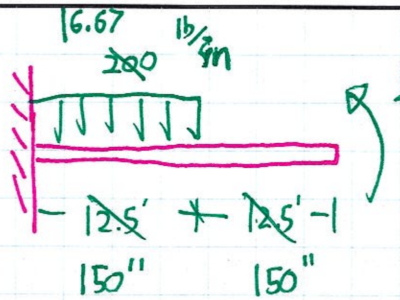
$$M = 100\langle x-12.5\rangle^2 - 100x^2 + 2500x - 14323$$



$$M_o \langle x - \tfrac{L}{2} \rangle^0$$

$$\frac{M_o}{1} \langle x - \tfrac{L}{2} \rangle^1$$

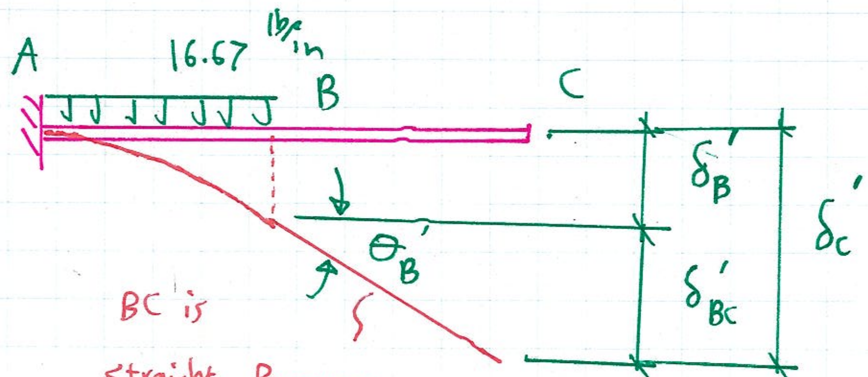
All original material presented was created by Michael S. Brackin, Ph.D., P.E. and is copyrighted 2020. No Part or portion of this material may be copied, transmitted or otherwise used without the express written consent of Michael S. Brackin, Ph.D., P.E.



~~1302~~ ft-lb
15624 in-lb

$E = 29000$ ksi
$I = 60\ in^4$

I



BC is straight Because section has no load

$$\delta_B' = -\frac{wL^4}{8EI} = \frac{-(16.67)(150)^4}{8(29E6\ psi)(60)} = -0.606''$$

$$\theta_B' = \frac{-wL^3}{6EI} = \frac{-(16.67)(150)^3}{6(29E6)(60)} = -0.00539\ \text{rad}$$

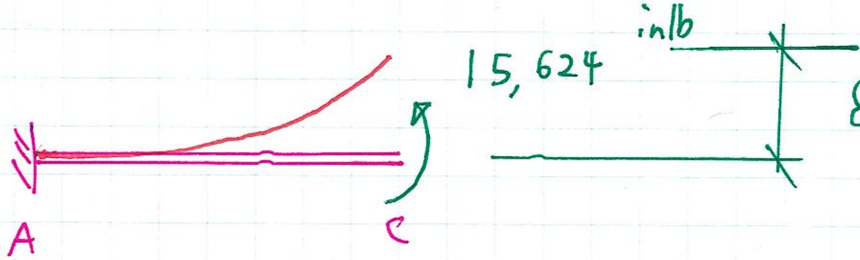
$y = mx + b$

$$\delta_C' = \underbrace{\theta_B'\left(\frac{L}{2}\right)}_{\delta_{BC}'} + \delta_B'$$

$$= -0.00539(150'') + (-0.606'') = -1.4145''$$

All original material presented was created by Michael S. Brackin, Ph.D., P.E. and is copyrighted 2020. No Part or portion of this material may be copied, transmitted or otherwise used without the express written consent of Michael S. Brackin, Ph.D., P.E.

II



$$\delta_C'' = \frac{ML^2}{2EI} = \frac{(15624 \text{ in-lb})(300)^2}{2(2966)(60)} = 0.404''$$

$$\delta_c = \delta_c' + \delta_c'' = -1.415 + 0.404'' = -1.01$$

All original material presented was created by Michael S. Brackin, Ph.D., P.E. and is copyrighted 2020. No Part or portion of this material may be copied, transmitted or otherwise used without the express written consent of Michael S. Brackin, Ph.D., P.E.