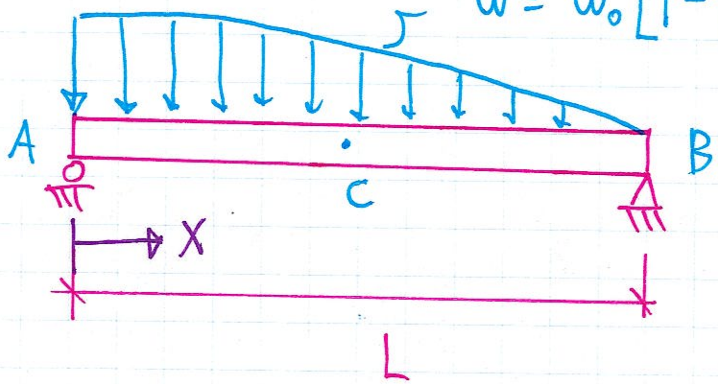
$[x=0,\ y=0]$

$[x=0,\ M=0]$

$$w = w_0\left[1 - \frac{x^2}{L^2}\right]$$

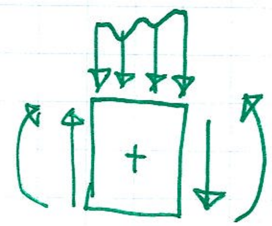


$[x=L,\ y=0]$

$[x=L,\ M=0]$

Determine (a) equation for deformation
(b) the slope at end A
(c) deflection at the middle

POS. SIGN CONV



a) $$\frac{dV}{dx} = -w = -w_0\left[1 - \frac{x^2}{L^2}\right] \qquad \text{"load"}$$

$$\frac{dM}{dx} = V = -w_0 x + \frac{w_0 x^3}{3L^2} + C_1 \qquad \text{"shear"}$$

$$EI\frac{d^2y}{dx^2} = M = \frac{-w_0 x^2}{2} + \frac{w_0 x^4}{12L^2} + C_1 x + C_2 \qquad \text{"Moment"}$$

y''

$$EI\frac{dy}{dx} = \frac{-w_0 x^3}{6} + \frac{w_0 x^5}{60L^2} + \frac{C_1 x^2}{2} + C_2 x + C_3 \qquad \text{"slope"}$$

$$EI\,y = -\frac{w_0 x^4}{24} + \frac{w_0 x^6}{360L^2} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4 \qquad \text{"deflection"}$$

BNDY. COND.

[x=0, M=0]

$$0 = -\frac{w_o}{2}\cancelto{0}{(0)^2} + \frac{w_o}{12L^2}\cancelto{0}{(0)^4} + \cancelto{0}{C_1(0)} + C_2 \Rightarrow C_2 = 0$$

[X=L, M=0]

$$0 = -\frac{w_o}{2}(L)^2 + \frac{w_o}{12\cancel{L^2}}(L)^{\cancelto{2}{4}} + C_1(L) \Rightarrow C_1 = \frac{5w_oL}{12}$$

[X=0, y=0]

$$EI(0) = \cancelto{0}{-\frac{w_o(0)^4}{24}} + \cancelto{0}{\frac{w_o(0)^6}{360L^2}} + \cancelto{0}{\frac{5w_oL}{12\cdot6}(0)^3} + \cancelto{0}{C_3(0)} + C_4 \Rightarrow C_4 = 0$$

[X=L, y=0]

$$EI(0) = -\frac{w_o L^4}{24} + \frac{w_o L^{\cancelto{4}{6}}}{360\cancel{L^2}} + \frac{5w_oL}{12\cdot6}\cancelto{4}{L^3} + C_3(L) + 0 \Rightarrow C_3 = \frac{-11w_oL^3}{360}$$

$$\boxed{y = \frac{1}{EI}\left[-\frac{w_o x^4}{24} + \frac{w_o x^6}{360 L^2} + \frac{5}{12} w_o L x^3 - \frac{11}{360} w_o L^3 x \right]}$$

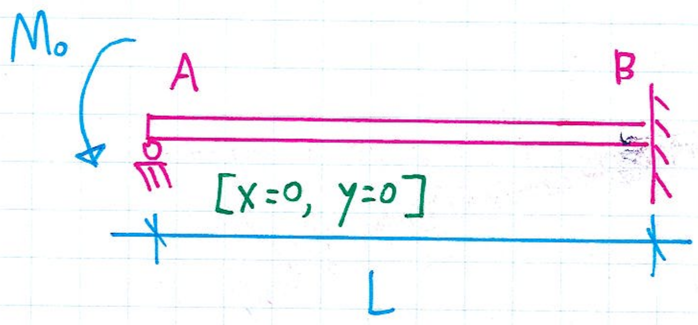
(b) @ "A", $x = 0$ $\Rightarrow$ most term fall out

$$\theta_A = \frac{dy}{dx} = \frac{C_3}{EI} = \boxed{\frac{-11 w_o L^3}{360 EI}}$$

(c)

@ "C", $x = \frac{L}{2}$

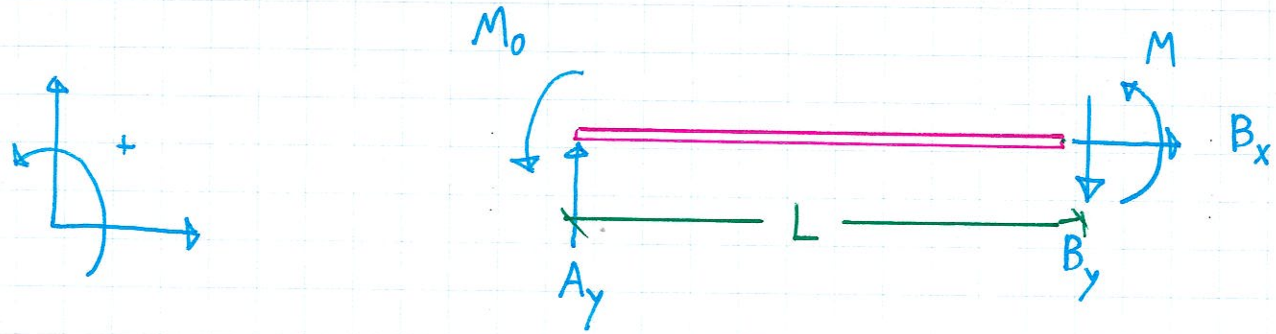
$$\boxed{y_c = \frac{1}{EI}\left[\frac{-w_o \left(\frac{L}{2}\right)^4}{24} + \frac{w_o \left(\frac{L}{2}\right)^6}{360 L^2} + \frac{5}{12} w_o L \left(\frac{L}{2}\right)^3 - \frac{11}{360} w_o L^3 \left(\frac{L}{2}\right) \right]}$$



$[x=L,\ y=0]$ Determine the reactions at A & B

$[x=L,\ \frac{dy}{dx}=0]$

FBD_{AB}



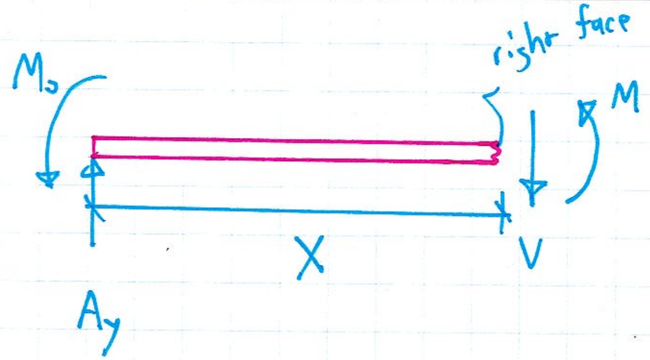
$\overset{+}{\rightarrow}\ \sum F_x = 0; \quad B_x = 0$

$+\uparrow\ \sum F_y = 0; \quad A_y = B_y$

$\circlearrowleft +\ \sum M_B = M + M_0 - A_y L = 0$

$\Rightarrow M = A_y L - M_0$

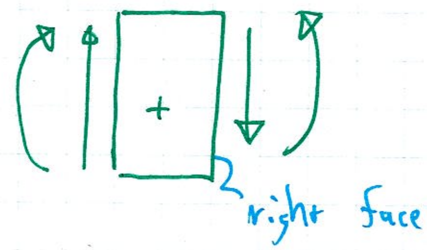
FBD



$$\overset{+}{\curvearrowleft} \sum_1 M_{cut} = M + M_o - A_y X = 0$$

$$\Rightarrow M(x) = A_y X - M_o$$

Sign Conv.



$$EI \frac{d^2y}{dx^2} = M(x) = A_y X - M_o$$

$$EI \frac{dy}{dx} = \frac{A_y}{2} X^2 - M_o X + C_1$$

$$EI\, y = \frac{A_y}{6} X^3 - \frac{M_o}{2} X^2 + C_1 X + C_2$$

BNDY. COND.

$$\left[x = L,\ \frac{dy}{dx} = 0\right]$$

$$EI(0) = \frac{A_y}{2}L^2 - M_o(L) + C_1 \implies C_1 = M_o L - \frac{A_y}{2}L^2$$

$$[x = 0,\ y = 0]$$

$$EI(0) = \frac{A_y}{6}(0)^3 - \frac{M_o}{2}(0)^2 + C_1(0) + C_2 \implies C_2 = 0$$

THUS,

$$EI\,y = \frac{A_y}{6}x^3 - \frac{M_o}{2}x^2 + \left(M_o L - \frac{A_y}{2}L^2\right)x$$

SOLVE for A_y; $[x = L,\ y = 0]$

$$EI(0) = \frac{A_y}{6}(L)^3 - \frac{M_o}{2}(L)^2 + M_o L^2 - \frac{A_y}{2}L^3$$

$$\underline{A_y = \frac{3}{2}\frac{M_o}{L}} \uparrow$$

$$M = A_y L - M_0 = \left[\frac{3}{2}\frac{M_0}{\cancel{L}}\right]\cancel{L} - M_0 = \frac{1}{2}M_0$$

$$B_y = A_y = \frac{3}{2}\frac{M_0}{L}$$

$$B_x = 0$$