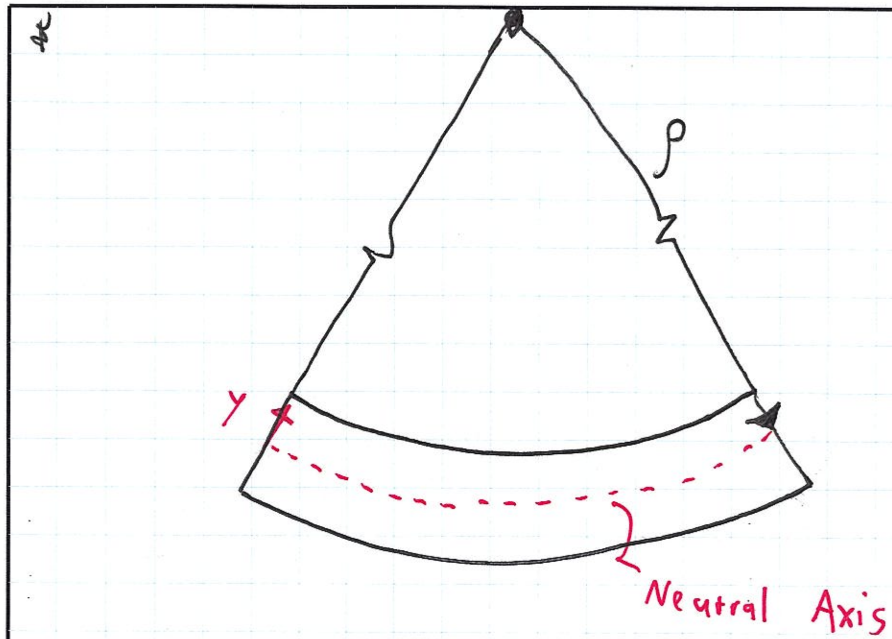

$$\frac{1}{\rho} = \frac{M}{EI}$$ "from bending theory"

$$\frac{1}{\rho} = \frac{\left(\frac{d^2y}{dx^2}\right)}{\cancel{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}} \rightarrow 1$$ assuming small deformations

$$\frac{1}{\rho} = \frac{d^2y}{dx^2}$$

$$\frac{M(x)}{EI} = \frac{d^2y}{dx^2} \Longrightarrow M(x) = \frac{d^2y}{dx^2} EI$$ "MOMENT"

$$\int M(x) = \int \frac{d^2y}{dx^2} EI = \frac{dy}{dx} EI$$ "slope"

$$\iint M(x) = \iint \frac{d^2y}{dx^2} EI = y \, EI$$ "deflection"

Integration ⇒ leads to constants of integration

figure out const. w/ bndy. conditions
- deflection @ a point
- slope @ a point

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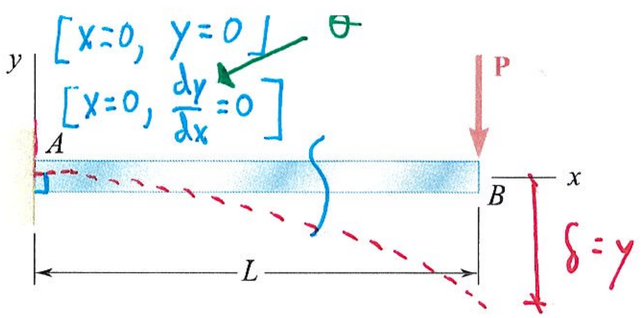


For the loading shown, determine

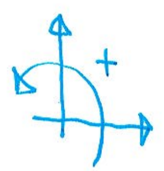
Part 1 out of 3

(a) the equation of the elastic curve for the cantilever beam AB,

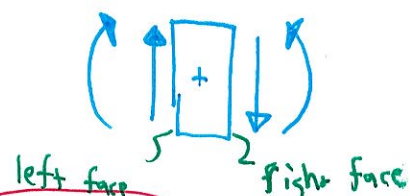
$$y = -\frac{Px^{\square}}{\square EI}(\square L - x)$$

next

EQUIL. SIGN CONV.

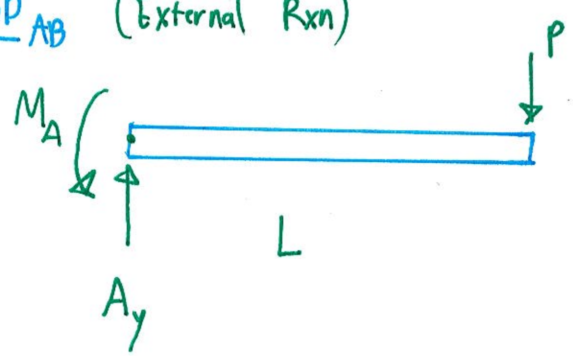


V&M SIGN CONV.



$$M(x) = EI\frac{d^2y}{dx^2}$$

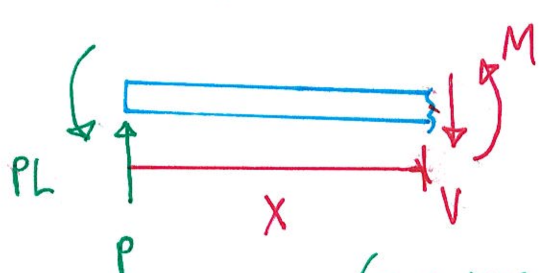
FBD$_{AB}$ (External Rxn)



$\overset{+\uparrow}{\sum F_y} = 0;$ $A_y = P$

$\overset{+\circlearrowleft}{\sum M_A} = 0;$ $M_A = PL$

FBD (Internal)



$(0 < x < L)$

$\overset{+\circlearrowleft}{\sum M_{cut}} = M - P(x) + PL = 0$

$\Rightarrow M = Px - PL$

$\underbrace{M(x)}_{\text{function of } x} = Px - PL$

$$M(x) = EI \frac{d^2y}{dx^2}$$

$$Px - PL = EI \frac{d^2y}{dx^2}$$

"MOMENT"

$$\int (Px - PL)\,dx = \int \left(EI \frac{d^2y}{dx^2}\right) dx$$

slope

$$P\left(\frac{x^2}{2} - Lx\right) + C_1 = EI \frac{dy}{dx}$$

$$\int \left[P\left(\frac{x^2}{2} - Lx\right) + C_1\right] dx = \int EI\left(\frac{dy}{dx}\right) dx$$

deflection

$$P\left(\frac{x^3}{6} - \frac{Lx^2}{2}\right) + C_1 x + C_2 = EI\, y$$

BNDY. COND.

$$P\left(\frac{x^2}{2} - Lx\right) + C_1 = EI\frac{dy}{dx}$$

$$\left[x=0,\ \frac{dy}{dx}=0\right]$$

$$P\left(\frac{0^2}{2} - L(0)\right) + \cancel{C_1} = EI(0) \qquad \underline{\underline{C_1=0}}$$

$$P\left(\frac{x^3}{6} - \frac{Lx^2}{2}\right) + \cancel{C_1}^{\,0}x + C_2 = EIy$$

$$[x=0,\ y=0]$$

$$P\left(\frac{0^3}{6} - \frac{L(0)^2}{2}\right) + 0(0) + \cancel{C_2} = EI(0) \qquad \underline{\underline{C_2=0}}$$

$$P\left(\frac{x^3}{6} - \frac{Lx^2}{2}\right) = EIy$$

$$\Rightarrow y = \frac{P}{EI}\left(\frac{x^3}{6} - \frac{Lx^2}{2}\right)$$

$$= \frac{Px^2}{EI}\left(\frac{x}{6} - \frac{L}{2}\right)$$

$$= \frac{Px^2}{EI}\left(\frac{-3L}{6} + \frac{x}{6}\right)$$

$$\boxed{y = \frac{-Px^2}{6EI}(3L - x)}$$